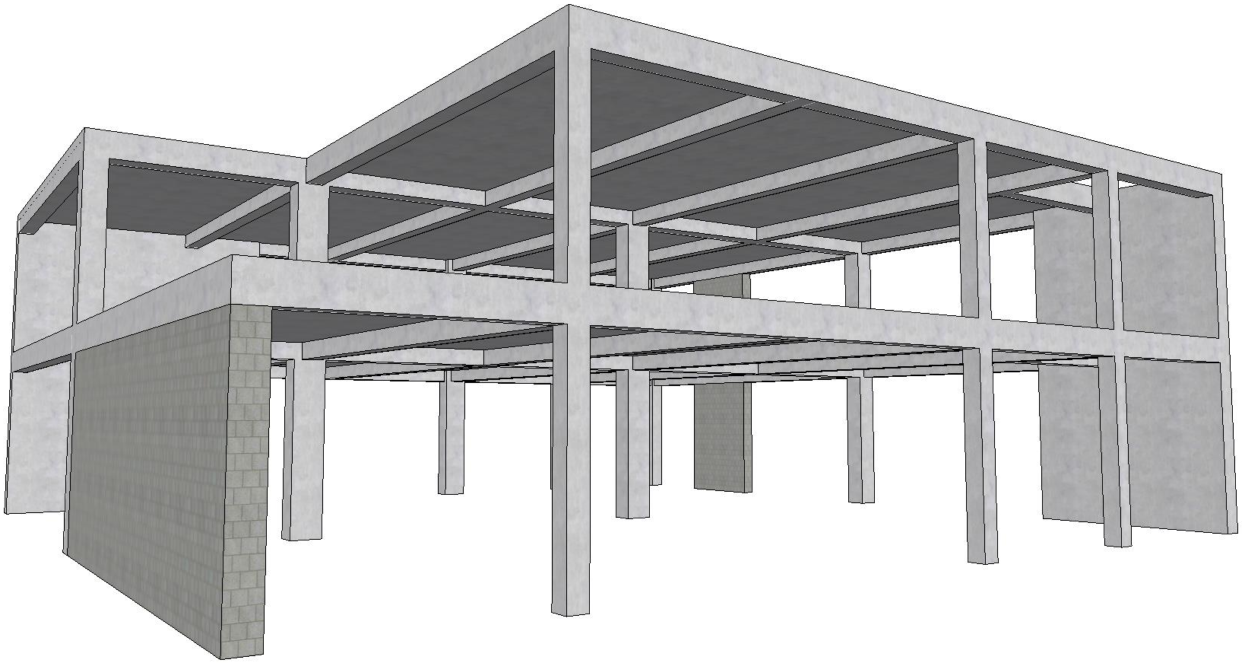
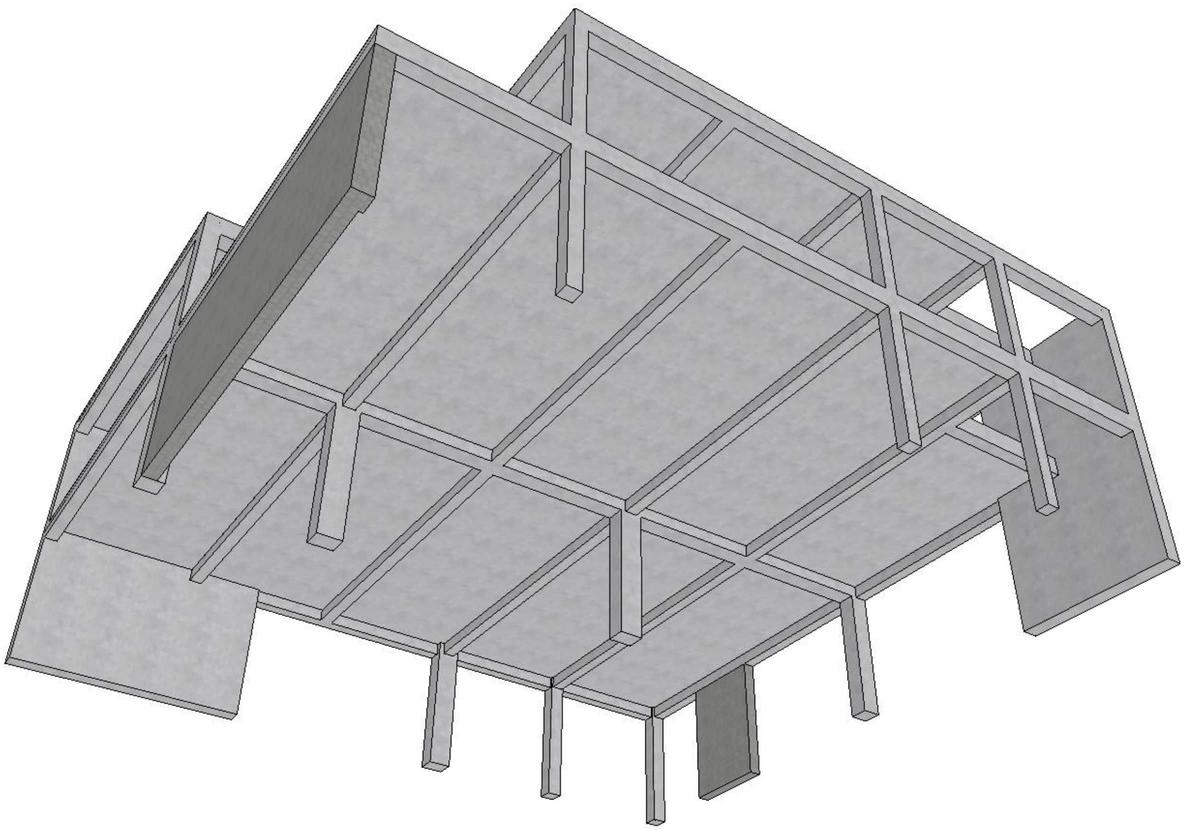
Reinforced Concrete Continuous Beam Analysis and Design (CSA A23.3-14)





Reinforced Concrete Continuous Beam Analysis and Design (CSA A23.3-14)

A structural reinforced concrete continuous beams at an intermediate building floor provides gravity load resistance for the applied dead and live loads.

The continuous beam along grid 3 is selected to demonstrate the analysis and design of continuous T-beams (structural analysis, flexural design, shear design, deflection checks) and the results of hand calculations are then compared with numerical analysis results obtained from the [spBeam](#) engineering software program.

Additionally, the boundary conditions for each T-beam (Grids 1 thru 6 and A thru C) are selected to demonstrate and explore in detail the actual interaction between the beam and the supporting members. Similar evaluation is performed using computer software to reflect recommended modeling procedures in [spBeam](#) to obtain the most accurate results.

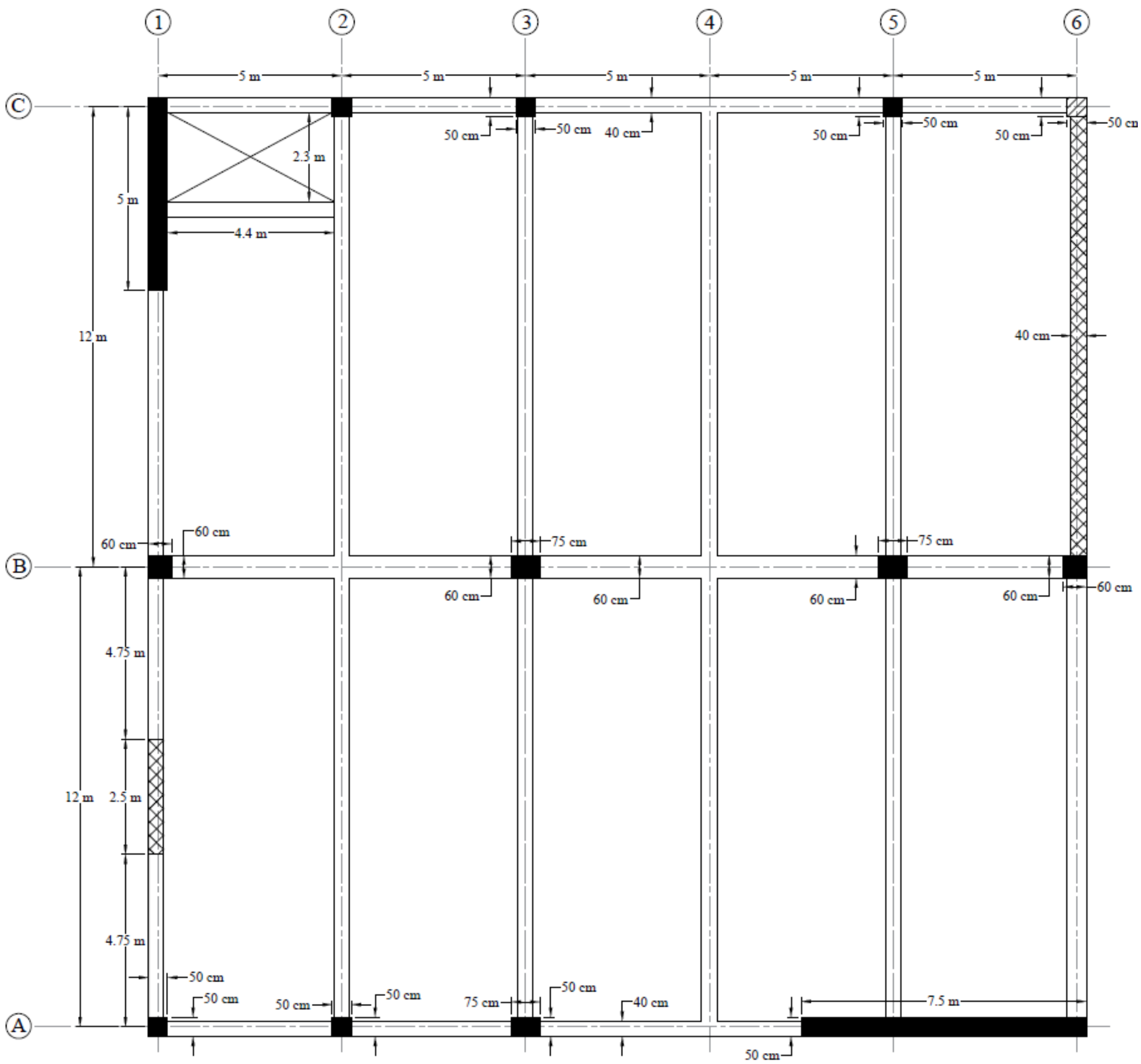


Figure 1 – Reinforced Concrete Continuous Beams at intermediate building floor

Contents

Code

Design of Concrete Structures (CSA A23.3-14) and Explanatory Notes on CSA Group standard A23.3-14 "Design of Concrete Structures"

Reference

CAC Concrete Design Handbook, 4th Edition, Cement Association of Canada

Reinforced Concrete Mechanics and Design, First Canadian Edition, 2000, James MacGregor and Michael Bartlett, Prentice Hall.

Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association

spBeam Engineering Software Program Manual v5.00, STRUCTUREPOINT, 2015

Design Data

f_c' = 25 MPa normal weight concrete (w_c = 24 kN/m^3)

f_y = 400 MPa

Superimposed dead load, SDL = 1 kN/m^2

Typical Floor Level, Live load, L_o = 2.5 kN/m^2

Typical Floor Level, Reduced Live load, L_o = 1.6 kN/m^2

Solution

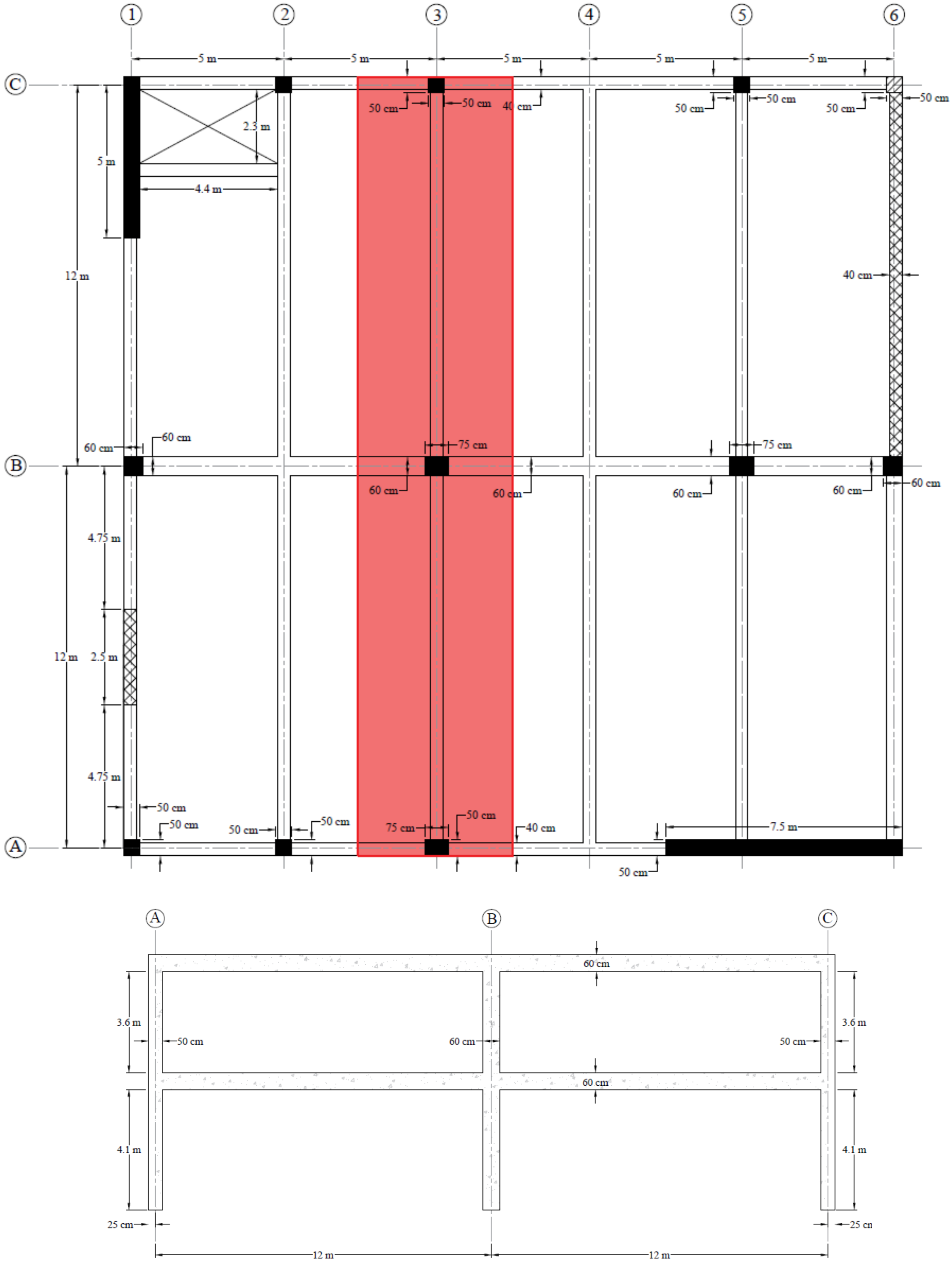


Figure 2 – Reinforced Concrete Continuous Beam (Grid 3)

1. Preliminary Member Sizing

Check the minimum beam depth requirement of ***CSA A23.3-14 (9.8.2.1)*** to waive deflection computations. Using the minimum depth for non-prestressed beams in ***Table 9.2***.

End Span: $$h = \frac{l_n}{18} = \frac{11,500}{18.5} = 639 \text{ mm}$$ ***CSA A23.3-14 (Table 9.2)***

Therefore, since h_{min} = 639 mm > h = 600 mm the preliminary beam depth does not satisfy the minimum depth requirement, and the beam deflection need to be checked.

2. Load and Load combination

For the factored Load

$$w_f = 1.25D + 1.5L$$ ***CSA A23.3-14 (Annex C, Table C.1a)***

$$D = 5\times(1+0.2\times24)+(0.4\times0.4\times24) = 32.84 \text{ kN/m}$$

$$L = 1.6\times5 = 8 \text{ kN/m}$$

$$w_u = 1.25\times32.84+1.5\times8 = 53.05 \text{ kN/m}$$

3. Structural Analysis by Moment Distribution

The T-beam will be analyzed by hand using the moment distribution method to determine design moment and shear values. Members stiffnesses, carry over factors COF, and fixed-end moments FEM for the beam and column members are determined as follows:

3.1. Flexural stiffness of beams, K_b

$$K_b = 4\times\frac{E_c I_b}{L_b} = 4\times\frac{25684\times1.52\times10^{10}}{12000} = 1.3\times10^{11} \text{ N.mm}$$

Where I_b is calculated for the beam as a T-beam section with b_{eff} equals to:

$$b_{eff} = \min\left\{\begin{array}{l} b+\left(\frac{1}{5}\times l\right) = 400+\frac{1}{5}\times12,000 = 2,800 \text{ mm} \\ b+12\times t_{flange} = 400+12\times200 = 2,800 \text{ mm} \\ b+2\times\frac{l_n}{8} = 400+\frac{1}{2}\times5,000 = 2,900 \text{ mm} \end{array}\right\} = 2,800 \text{ mm}$$ ***CSA A23.3-14 (10.3.3)***

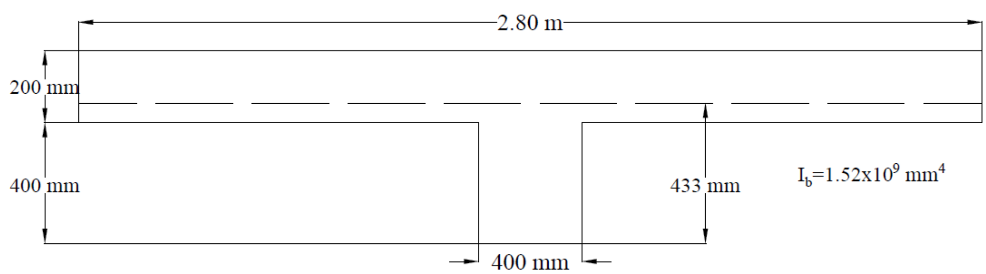


Figure 3 – Moment of inertia calculation for T-beam section

$$E_c = (3,300\sqrt{f_c^{'}} + 6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5} = (3,300\sqrt{25} + 6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 25,684 \text{ MPa}$$ **CSA A23.3-14(8.6.2.2)**

Carry-over factor *COF* = 0.5

Fixed-end moment, $FEM = \frac{w_f \times L_b^2}{12} = \frac{53.05 \times 12,000^2}{12} = 636.6 \text{ kN.m}$

3.2. Flexural stiffness of column members, K_c

For the Top Exterior Column:

$COF_{c,top} = 0.50$

$$K_{c,top} = 4 \times \frac{E_c I_c}{L_c} = 4 \times \frac{25,684 \times 5.21 \times 10^9}{4200} = 1.27 \times 10^{11} \text{ N.mm}$$

Where $I_c = \frac{c_2 \times c_1^3}{12} = \frac{500 \times 500^3}{12} = 5.21 \times 10^9 \text{ mm}^4$

$$E_c = (3,300\sqrt{f_c^{'}} + 6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5} = (3,300\sqrt{25} + 6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 25,684 \text{ MPa}$$ **CSA A23.3-14(8.6.2.2)**

L_c = 4200 mm

For the Bottom Exterior Column:

$COF_{c,bottom} = 0.50$

$$K_{c,bottom} = 4 \times \frac{E_c I_c}{L_c} = 4 \times \frac{25,684 \times 5.21 \times 10^9}{4400} = 1.22 \times 10^{11} \text{ N.mm}$$

Where $I_c = \frac{c_2 \times c_1^3}{12} = \frac{500 \times 500^3}{12} = 5.21 \times 10^9 \text{ mm}^4$

L_c = 4400 mm

For the Top Interior Column:

$COF_{c,top} = 0.50$

$$K_{c,top} = 4 \times \frac{E_c I_c}{L_c} = 4 \times \frac{25,684 \times 1.35 \times 10^{10}}{4200} = 3.30 \times 10^{11} \text{ N.mm}$$

Where $I_c = \frac{c_2 \times c_1^3}{12} = \frac{750 \times 600^3}{12} = 1.35 \times 10^{10} \text{ mm}^4$

L_c = 4200 mm

For the Bottom Interior Column:

$COF_{c,bottom} = 0.50$

$$K_{c,bottom} = 4 \times \frac{E_c I_c}{L_c} = 4 \times \frac{25,684 \times 1.35 \times 10^{10}}{4400} = 3.15 \times 10^{11} \text{ N.mm}$$

Where $I_c = \dfrac{c_2 \times c_1^3}{12} = \dfrac{750 \times 600^3}{12} = 1.35 \times 10^{10}\ \text{mm}^4$

L_c = 4400 mm

3.3. Beam joint distribution factors, DF

At exterior joint,

$$DF = \frac{1.30 \times 10^{11}}{(1.30 \times 10^{11} + 1.27 \times 10^{11} + 1.22 \times 10^{11})} = 0.340$$

At interior joint,

$$DF = \frac{1.30 \times 10^{11}}{(1.30 \times 10^{11} + 1.30 \times 10^{11} + 3.30 \times 10^{11} + 3.15 \times 10^{11})} = 0.140$$

COF for beams =0.5

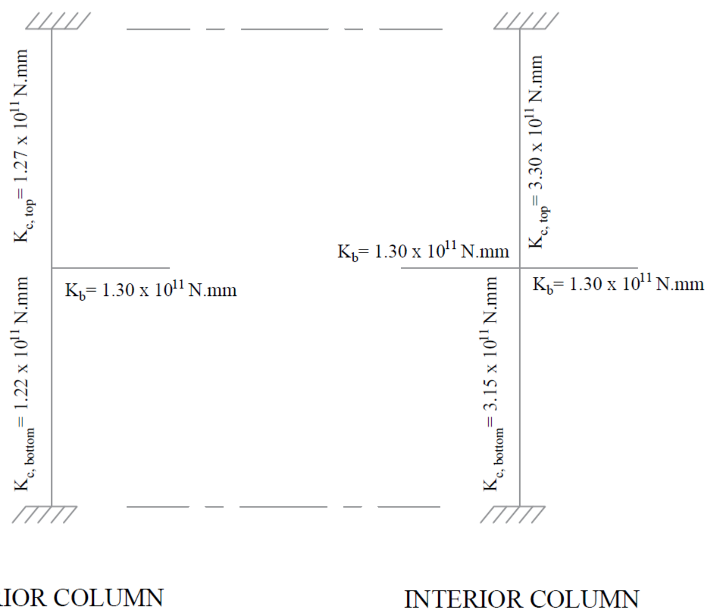


Figure 4 – beams and Columns Stiffnesses

3.4. Moment Distribution

When the live load is uniformly distributed and does not exceed three-quarters of the specified dead load or the nature of the live load is such that all panels will be loaded simultaneously, the maximum factored moments may be assumed to occur at all sections with full factored live load on the entire slab system.

CSA A23.3-14 (13.8.4.2)

$$\frac{L}{D} = \frac{1.6}{(1 + 0.6 \times 24)} = 0.16 < \frac{3}{4}$$

Moment Distribution computations are shown in the following Table. Counterclockwise rotational moments acting on the member ends are taken as positive. Positive span moments are determined from the following equation:

$$M_{u,midspan} = M_o - \frac{(M_{uL} + M_{uR})}{2}$$

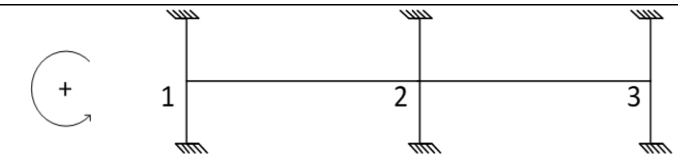
Where M_o is the moment at the midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at the midspan, but its value is close to that midspan for this example (see results comparison section).

Positive moment in span 1-2:

$$M_f = 53.05 \times \frac{12^2}{8} - \frac{418.1 + 745.9}{2} = 372.9 \text{ kN.m}$$

Table 1 - Moment Distribution for Continuous Beam



Joint	1	2		3
Member	1-2	2-1	2-3	3-2
DF	0.34	0.14	0.14	0.34
COF	0.50	0.50	0.50	0.50
FEM	636.6	-636.6	636.6	-636.6
Dist	-218.5	0.0	0.0	218.5
CO	0.0	-109.3	109.3	0.0
Dist	0.00	0.00	0.00	0.00
M, kN.m	418.1	-745.9	745.9	-418.1
Midspan M, kN.m	372.9		372.9	

Except when approximate values for bending moments are used, the negative moments at the supports of continuous flexural members calculated by elastic analysis for any assumed loading arrangement may each be increased or decreased by not more than (30 – 50c / d)%, but not more than 20%, and the modified negative moments shall be used for calculating the moments at sections within the spans. ***CSA A23.3-14 (9.2.4)***

The moment redistribution is often utilized for the investigation of existing structures for conditions such as change of use, additional loading, or verifying adequacy for the latest design code. In these conditions, any reserve capacity from existing reinforcement layout at mid-span (or support) of a span may be utilized to compensate for the inadequacy of the support (or mid-span) of the same span.

The moment redistribution can also be utilized in the design of a new structure. One such example of its application may help reduce the negative moment at an interior support and corresponding top reinforcement while increasing the positive moment at mid-span. The advantage of this may be the alleviation of the congestion of rebar at support top regions.

The calculation of moment redistribution is a tedious process especially while considering live load patterning. The procedure gets far more complicated if point loads or partial line loads are present. The spBeam software program performs the moment redistribution calculations with speed and accuracy.

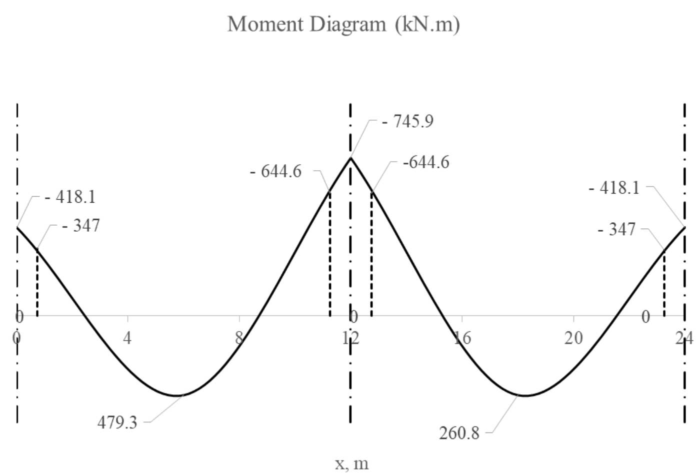
This example does not cover the moment redistribution. However, a detailed demonstration of this method can be found in "Continuous Beam Design with Moment Redistribution (CSA A23.3-14)" example.

3.5. Factored moments used for Design

Positive and negative factored moments for the continuous beam are plotted in the following Figure. The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than 0.175 *l* from the centers of supports.

CSA A23.3-14 (13.8.5.1)

$$\frac{500}{1000} = 0.5 \text{ m} < 0.175 \times 12 = 2.1 \text{ m (use face of supporting location for interior column)}$$



$$\frac{500}{1000} = 0.5 \text{ m} < 0.175 \times 12 = 2.1 \text{ m (use face of supporting location for interior column)}$$

Figure 5 - Positive and Negative Design Moments for the Continuous Beam

4. Flexural Design

For this beam, the moment at the exterior face of the interior support governs the design as shown in the previous Figure.

Calculate the required reinforcement to resist the interior support negative moment:

$$M_f = 664.6 \text{ kN.m}$$

Use 30M bars with 30 mm concrete cover per ***CSA A23.3-14 (Table 17)***. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, *d*, is calculated below:

$$d = 600 - (30 + 0.5 \times 30) = 555 \text{ mm}$$

In this example, *jd* is assumed equal to *0.728d*. The assumption will be verified once the area of steel in finalized.

$$jd = 0.728 \times d = 0.728 \times 555 = 404 \text{ mm}$$

$$b = 400 \text{ mm}$$

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_f}{\varphi_s f_y jd} = \frac{664.6 \times 10^6}{0.85 \times 400 \times 404} = 4,692 \text{ mm}^2$$

$$\alpha_1 = 0.85 - 0.0015 f_c^{'} = 0.81 > 0.67$$ ***CSA A23.3-14 (10.1.7)***

$$\beta_1 = 0.97 - 0.0025 f_c^{'} = 0.91 > 0.67$$ ***CSA A23.3-14 (10.1.7)***

Recalculate '*a*' for the actual A_s = 4,692 mm^2: $a = \dfrac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \dfrac{0.85 \times 4,692 \times 400}{0.65 \times 0.81 \times 25 \times 400} = 302 \text{ mm}$

$$c = \frac{a}{\beta_1} = \frac{302}{0.81} = 333 \text{ mm}$$

The tension reinforcement in flexural members shall not be assumed to reach yield unless:

$$\frac{c}{d} \le \frac{700}{700 + f_y}$$ ***CSA A23.3-14 (10.5.2)***

$$\frac{333}{555} = 0.60 \le 0.64$$

$$jd = d - {}^{a}\!/_{2} = 0.728\ d$$

Therefore, the assumption that tension reinforcements will yield and *jd* equals to *0.728d* is valid.

The minimum reinforcement shall not be less than

$$A_{s,\min} = \frac{0.2 \times \sqrt{f_c^{'}}}{f_y} \times b_t \times h = \frac{0.2\sqrt{25}}{400} \times 1,000 \times 600 = 1,500 \text{ mm}^2$$ ***CSA A23.3-14 (10.5.1.2)***

Where b_t is the width of the tension zone of the section considered. For T-beams with the flange in tension, b_t need not exceed 1.5bw for beams with a flange on one side of the web or 2.5b_w for beams with a flange on both sides of the web. ***CSA A23.3-14 (10.5.1.2)***

Provide 7 – 30 M bars:

$$A_{s,prov} = 7 \times 700 = 4,900 \text{ mm}^2 > A_{s,req} = 4,692 \text{ mm}^2$$

The reinforcement is selected based on the closest $A_{s,provided}$ to $A_{s,required}$. However, moment redistribution can be used to reduce the A_s (section 3.4). Furthermore, other bar size and detailing options can be selected (e.g. 10 – 25M in one or two layers) to overcome any construction issues or preferences.

<u>Maximum spacing allowed:</u>

$$s = \max\begin{Bmatrix} 3t_{slab} \\ 500 \text{ mm} \end{Bmatrix} = 500 \text{ mm}$$ ***CSA A23.3-14 (7.4.1.2)***

Check the requirement for distribution of flexural reinforcement to control flexural cracking:

$$z = f_s(d_c A)^{1/3}$$ ***CSA A23.3-14 (10.6.1)***

Use $f_s = 0.6 f_y = 240$ MPa ***CAC Concrete Design Handbook – 4th Edition (2.3.2)***

$$A = 2yb_w/8 = 2\times 45\times 400/8 = 4,500 \text{ mm}^2$$

$$d_c = 30 + 11.3 + \frac{35.7}{2} = 59 \text{ mm}$$

$$z = 240(59\times 4,500)^{1/3} = 15,425 \text{ N/mm} < 30,000 \text{ N/mm}$$

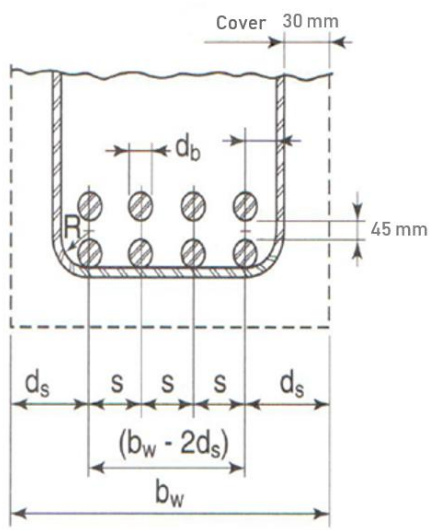


Figure 6 - Maximum number of bars in beams

Where flanges are in tension, part of the flexural tension reinforcement shall be distributed over an overhanging flange width equal to 1/20 of the beam span, or the width specified in Clause 10.3 of CSA A23.3-14, whichever is smaller. The area of this reinforcement shall be not less than 0.004 times the gross area of the overhanging flange. ***CSA A23.3-14 (10.5.3.1)***

$$b = \min\begin{Bmatrix} b_{eff} = 2,800 \text{ mm} \\ 400 + 2\times\frac{l}{20} = 400 + 2\times\frac{12,000}{20} = 1,600 \text{ mm} \end{Bmatrix} = 1,600 \text{ mm}$$

Check if $s_{provided}$ is greater than the minimum center to center spacing, s_{min} where

$$s_{min} = \max\begin{Bmatrix} d_b \\ 1.4\times \max.\ agg. \\ 30 \text{ cm} \end{Bmatrix}$$ ***CSA A23.3-14 (Annex A 6.6.5.2)***

Where the maximum aggregate size is 1.9 cm

$$s_{\min} = \max\begin{Bmatrix} 1.4\times 30 \\ 1.4\times 19 \\ 30 \text{ mm} \end{Bmatrix} = 42 \text{ mm}$$

Since the spacing provided is greater than 42 mm Therefore, 7 – 30M bars are <u>***o.k.***</u>

All the values in the following table are calculated based on the procedure outlined above.

Table 2 – Reinforcing Design Summary			
	End Span		
	Exterior Negative	Positive	Interior Negative
Design Moment, M_f (kN.m)	340.0	372.9	644.6
Effective depth, d (mm)	555	555	555
$A_{s,req}$ (mm^2)	2092	2014	4692
$A_{s,min}$ (mm^2)	1500	600	1500
Reinforcement	5 - 30M*	3 - 30M	7 - 30M
Spacing provided (mm)	47.5	125	47.5
* Number of bars governed by maximum allowable spacing			

5. Shear Design

From the following Figure, the shear value in end span at face of the interior support governs.

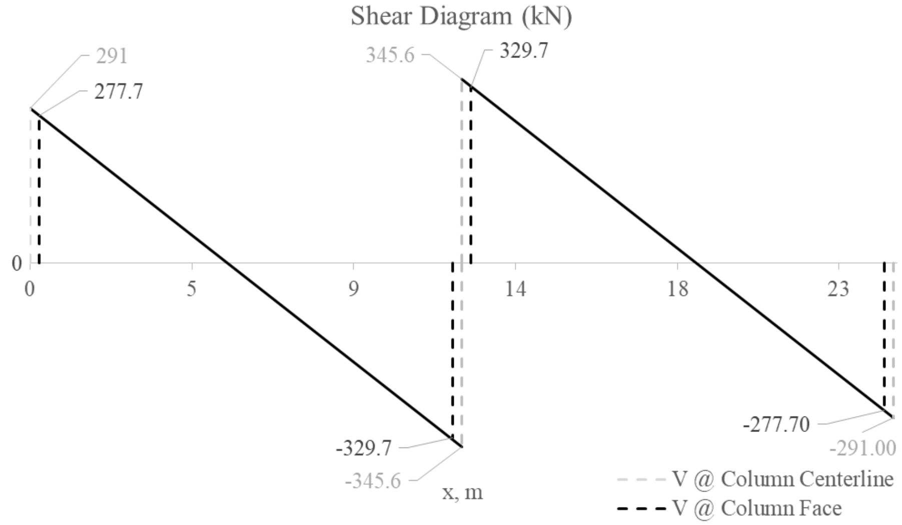


Figure 7 - Positive and Negative Design Shears for the Continuous Beam

$$V_f = \frac{-745.9 + 418.1 + \dfrac{w_f \times l}{2}}{l} = -345.6 \text{ kN}$$

The design shear at a distance, d_v, away from the face of support,

$$d_v = Max\ (0.9d, 0.72h) = Max\ (0.9 \times 555, 0.72 \times 600) = 499.5 \text{mm}$$ **CSA A23.3-14 (3.2)**

$$V_f = 345.6 - 53.05 \times \left(\frac{300 + 499.5}{1000}\right) = 305.85 \text{ kN}$$

The factored shear resistance shall be determined by

$$V_r = V_c + V_s + V_p = V_c$$ **CSA A23.3-14 (Eq. 11.4)**

However, V_r shall not exceed

$$V_{r,\max} = 0.25\varphi_c f_c^{'} b_w d_v + V_p$$ **CSA A23.3-14 (Eq. 11.5)**

$$V_{r,\max} = 0.25 \times 0.65 \times 25 \times 400 \times 499.5 / 1000 = 811.7 \text{ kN} \rightarrow \therefore \text{section is adequate}$$

Shear strength provided by concrete

$$V_c = \varphi_c \lambda \beta \sqrt{f_c^{'}} b_w d_v$$ **CSA A23.3-14 (Eq. 11.5)**

$$\beta = 0.18$$ **CSA A23.3-14 (11.3.6.3)**

$$V_c = 0.65 \times 1 \times 0.18 \times \sqrt{25} \times 400 \times \frac{499.5}{1,000} = 116.9 \text{ kN}$$

Since $V_f > V_c$, shear reinforcement is required.

Try 10M, two-leg stirrups (A_v = 200 mm^2).

The nominal shear strength required to be provided by shear reinforcement is

$V_s = Vf - V_c = 305.85 - 117.4 = 188.46 \text{ kN}$

$\left(\dfrac{A_v}{s}\right)_{req} = \dfrac{V_f - V_c}{\phi \times f_{yt} \times d_v \times \cot\theta} = \dfrac{188.97 \times 1000}{0.85 \times 400 \times 499.5 \times \cot 35^\circ} = 0.78 \text{ mm}^2/\text{mm}$ **CSA A23.3-14 (11.3.5.1)**

Where $\theta = 35^\circ$ **CSA A23.3-14 (11.3.6.2)**

$s_{req} = \dfrac{A_v}{\left(\dfrac{A_v}{s}\right)_{req}} = \dfrac{200}{0.78} = 256 \text{ mm}$

$\left(\dfrac{A_v}{s}\right)_{\min} = \dfrac{0.06 \times \sqrt{f_c^{'}} \times b_w}{f_{yt}}$ **CSA A23.3-14 (11.2.8.2)**

$\left(\dfrac{A_v}{s}\right)_{\min} = \dfrac{0.06 \times \sqrt{25} \times 400}{400} = 0.30 \text{ mm}^2/\text{mm}$

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per **CSA A23.3-14 (11.3.8)**.

$0.125 \lambda \varphi_c f_c^{'} b_w d_v = 405.8 < V_f$ **CSA A23.3-14 (11.3.8.3)**

Therefore, maximum stirrup spacing shall be the smallest of $0.7d_v$ and *600 mm*. **CSA A23.3-14 (11.3.8.1)**

$s_{\max} = lesser\ of \begin{bmatrix} 0.7d_v \\ 600 \text{ mm} \end{bmatrix} = lesser\ of \begin{bmatrix} 0.7 \times 449.5 \\ 600 \text{ mm} \end{bmatrix} = lesser\ of \begin{bmatrix} 350 \text{ mm} \\ 600 \text{ mm} \end{bmatrix} = 350 \text{ mm}$

Use 10M @ 250 mm stirrups

$Vf = \dfrac{\phi_s \times A_v \times f_y \times d_v \times \cot\theta}{s} + V_c$ **CSA A23.3-14 (11.3.3 and 11.3.5.1)**

$V_r = \dfrac{0.85 \times 200 \times 400 \times 499.5 \times \cot 35^\circ}{250 \times 1000} + 116.9 = 194 + 116.9 = 310.92 \text{ kN} > V_f = 305.85 \text{ kN}$ **o.k.**

Compute where V_f is equal to V_c, and the stirrups can be stopped **CSA A23.3-14 (11.2.8.1)**

$x = \dfrac{V_f - V_c}{V_f} \times \dfrac{l}{2} = \dfrac{305.85 - 116.9}{305.85} \times \dfrac{12}{2} = 3.7 \text{ m}$

At interior end of the exterior span, use 15 – 10M @ 250 mm o.c., Place 1st stirrup 125 mm from the face of the column.

6. Deflection Control (Serviceability Requirements)

Since the preliminary beam depth did not meet minimum depth requirement, the deflection calculations are required. The calculation of immediate and time-dependent deflections are covered in detail in this section for illustration and comparison with spBeam model results for continuous T-beam.

6.1. Immediate (Instantaneous) Deflections

Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D+L_{Full}$) is used to obtain immediate deflections of the continuous T-beam in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests.

The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the beam. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \le I_g$$ **CSA A23.3-14 (Eq. 9.1)**

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a. For conventionally reinforced (nonprestressed) members, the effective moment of inertia, I_e, shall be calculated by Eq. (9.1) unless obtained by a more comprehensive analysis.

Unless deflections are determined by a more comprehensive analysis, immediate deflection shall be computed using elastic deflection equations using the effective moment of inertia in Eq. 9.1 in CSA A23.3-14.

CSA A23.3-14 (9.8.2.3)

For continuous prismatic members, the effective moment of inertia may be taken as the weighted average of the values obtained from following equation for the critical positive and negative moment sections.

CSA A23.3-14 (9.8.2.4)

$I_{e,avg} = 0.85\, I_{em} + 0.15\, I_{ec}$ for one end continuous **CSA A23.3-14 (Eq. 9.3)**

$I_{e,avg} = 0.70\, I_{em} + 0.15\,(I_{e1} + I_{e2})$ for two ends continuous **CSA A23.3-14 (Eq. 9.4)**

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously. The following Figure shows the maximum moments for the total service load level.

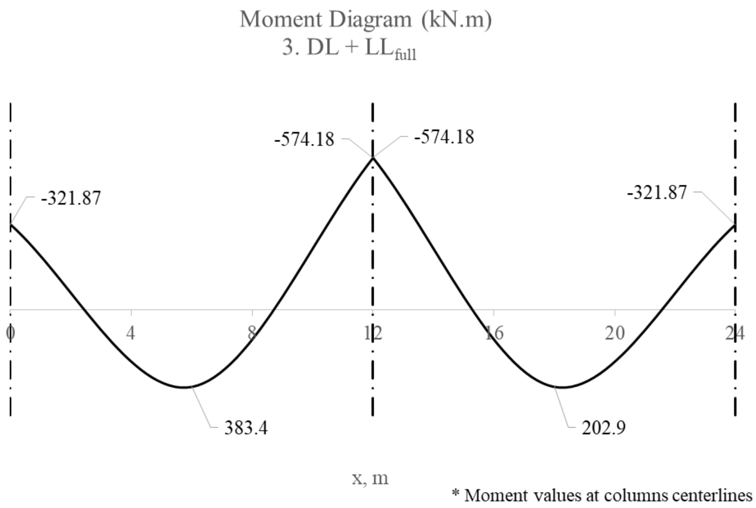


Figure 8 – Maximum Moments for the Total Service Load Level

For the negative moment region adjacent to the internal support:

The following calculations are based on considering the negative moment regions as rectangular sections. For T-beam, a T-shaped section can also be used to calculate I_{cr} in the negative moment regions. This approach will impact the calculations of I_g and M_{cr} where it has a slight impact on the instantaneous deflection results since the averaged effective moment of inertia is predominantly dependent on the properties of the mid-span region as can be seen later in this example. The hand calculations are continued with rectangular sections considered at the negative moment regions. A comparison between the two approaches is presented in "Design Results Comparison and Conclusions" section.

M_{cr} = cracking moment.

$$M_{cr} = \frac{f_r I_g}{Y_t} = \frac{(3.00/2)\times(7.2\times10^9)}{300}\times10^{-6} = 36 \text{ kN.m}$$ ***CSA A23.3-14 (Eq. 9.2)***

f_r should be taken as half of Eq. 8.3 in CSA A23.3-14 ***CSA A23.3-14 (9.8.2.3)***

f_r = Modulus of rapture of concrete.

$$f_r = 0.6\lambda\sqrt{f_c'} = 0.6\times1.0\times\sqrt{25} = 3.00 \text{ MPa}$$ ***CSA A23.3-14 (Eq.8.3)***

I_g = Moment of inertia of the gross uncracked concrete section

$$I_g = \frac{b\times h^3}{12} = \frac{400\times600^3}{12} = 7.20\times10^9 \text{ mm}^4$$

$$y_t = \frac{h}{2} = \frac{600}{2} = 300 \text{ mm}$$

I_{cr} = moment of inertia of the cracked section transformed to concrete.

CAC Concrete Design Handbook 4th Edition (5.2.3)

The exterior span near the interior support is reinforced with 7 – 30M bars.

E_c = Modulus of elasticity of concrete.

$$E_c = (3{,}300\sqrt{f_c^{'}} + 6{,}900)\left(\frac{\gamma_c}{2{,}300}\right)^{1.5} = (3{,}300\sqrt{25} + 6{,}900)\left(\frac{2{,}447}{2{,}300}\right)^{1.5} = 25{,}684 \text{ MPa}$$ ***CSAA23.3-14(8.6.2.2)***

$$n = \frac{E_s}{E_c} = \frac{200{,}000}{25{,}684} = 7.79$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$B = \frac{b}{n\,A_s} = \frac{400}{7.79\times(7\times700)} = 0.01 \text{ mm}^{-1}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$kd = \frac{\sqrt{2dB+1}-1}{B} = \frac{\sqrt{2\times555\times0.01+1}-1}{0.01} = 243.7 \text{ mm}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d-kd)^2$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{400\times243.7^3}{3} + 7.79\times(7\times700)\times(555-243.7)^2 = 5.63\times10^9 \text{ mm}^4$$

<u>For the exterior span (span with one end continuous) with total service load level:</u>

$$I_{ec} = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \text{, since } M_{cr} = 36 \text{ kN.m} < M_a = 461.71 \text{ kN.m}$$ ***CSA A23.3-14 (Eq. 9.1)***

Where I_e^- is the effective moment of inertia for the critical negative moment region (near the support).

As mentioned earlier, midspan stiffness (including the effect of cracking) has a dominant effect on deflections, midspan region is heavily represented in calculation of I_e and this is considered satisfactory in approximate deflection calculations. The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.85\,I_{em} + 0.15\,I_{ec} \text{ for one end continuous}$$ ***CSA A23.3-14 (Eq. 9.3)***

$$I_{e,avg} = 0.85\times\left(3.01\times10^9\right) + 0.15\times\left(5.63\times10^{10}\right) = 3.4\times10^9 \text{ mm}^4$$

Where:

I_{e1} = The effective moment of inertia for the critical negative moment section at end 1 of continuous beam span.

I_{e2} = The effective moment of inertia for the critical negative moment section at end 2 of continuous beam span.

The following Table provides a summary of the required parameters and calculated values needed for deflection calculation.

Table 3 – Averaged Effective Moment of Inertia Calculations (**<u>Rectangular</u>** Sections at Negative Moment Regions)

Span	zone	I_g, mm^4 (×10^9)	I_{cr}, mm^4 (×10^9)	M_a, kN.m			M_{cr}, kN.m	I_e, mm^4 (×10^9)			$I_{e,avg}$, mm^4(×10^9)		
				D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}	D	D + LL_{Sus}	D + L_{full}
Ext	Left	7.20	4.48	-258.82	-258.82	-321.87	-36.00	---	---	---	3.40	3.40	3.35
	Midspan	15.20	2.87	235.19	235.19	292.48	52.62	3.01	3.01	2.94			
	Right	7.20	5.63	-461.71	-461.71	-574.18	-36.00	5.63	5.63	5.63			

Table 4 – Averaged Effective Moment of Inertia Calculations (**T-Shaped** Sections at Negative Moment Regions)													
Span	zone	I_g, mm^4 ($\times10^9$)	I_{cr}, mm^4 ($\times10^9$)	M_a, kN.m			M_{cr}, kN.m	I_e, mm^4 ($\times10^9$)			$I_{e,avg}$, mm^4($\times10^9$)		
				D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}	D	D + LL_{Sus}	D + L_{full}
Ext	Left	15.20	4.48	-258.82	-258.82	-321.87	-136.80	---	---	---	3.44	3.44	3.37
	Midspan	15.20	2.87	235.19	235.19	292.48	52.62	3.01	3.01	2.94			
	Right	15.20	5.63	-461.71	-461.71	-574.18	-136.80	5.88	5.88	5.76			

After obtaining the averaged effective moment of inertia, the maximum span deflections for the continuous beam can be obtained from anyone of the selected three procedures, other procedures can be used too.

For the case were the negative moment regions are considered as rectangular sections:

6.1.1. PCA Superposition Procedure:

PCA Notes on ACI 318-11

Flat Plate Design Example

For exterior span - service total load case:

In order to be consistent with the CSA A23.3 methods, the clear span length is used in all method for the calculation of the deflection.

$$\Delta_{fixed} = \frac{w \times l_n^{\,4}}{384 \times E_c \times I_{e,avg}}$$

Where:

Δ_{fixed} = Deflection of beam assuming fixed end condition.

w = 37 + 3.84 = 40.84 kN/m

$I_{e,avg}$ = The averaged effective moment of inertia = 3.40×10^9 mm^4

$$E_c = (3{,}300\sqrt{f_c^{'}} + 6{,}900)\left(\frac{\gamma_c}{2{,}300}\right)^{1.5} = (3{,}300\sqrt{25} + 6{,}900)\left(\frac{2{,}447}{2{,}300}\right)^{1.5} = 25{,}684 \text{ MPa}$$

CSA A23.3-14(8.6.2.2)

$$\Delta_{fixed} = \frac{40.84 \times 11{,}450^4}{384 \times 25{,}684 \times 3.35 \times 10^9} = 21.27 \text{ mm}$$

$$\theta_L = \frac{M_{net,L}}{\Sigma K_c}$$

Where:

θ_L = Rotation of the span left support.

$M_{net,L}$ = 321.87 kN.m = Net negative moment of the left support.

$\sum K_c$ = column stiffness = (1.27+1.22) × 10^{11} N.mm = 2.49 × 10^{11} N.mm (calculated previously).

$$\theta_L = \frac{321.87 \times 10^6}{2.49 \times 10^{11}} = 1.29 \times 10^{-3} \text{ rad}$$

$$\Delta\theta_L = \theta_L\left(\frac{l}{8}\right)\left(\frac{I_g}{I_{e,avg}}\right)$$

Where:

$\Delta\theta_L$ = Midspan deflection due to rotation of left support.

$$\Delta\theta_L = 1.29\times10^{-3}\times\frac{11450}{8}\times\frac{7.20\times10^9}{3.35\times10^9} = 3.98 \text{ mm}$$

$$\theta_R = \frac{M_{net,R}}{\Sigma K_c} = \frac{0}{\Sigma K_c} = 0 \text{ rad}$$

$$\Delta\theta_R = \theta_R\left(\frac{l}{8}\right)\left(\frac{I_g}{I_{e,avg}}\right) = 0 \text{ mm}$$

$$\Delta_{Total} = \Delta_{fixed} + \Delta\theta_R + \Delta\theta_L$$

$$\Delta_{Total} = 21.27 + 3.98 + 0 = 25.25 \text{ mm}$$

Following the same procedure for dead service load level

$$\Delta_{DL} = 16.82 + 0.0 + 3.15 = 19.97 \text{ mm}$$

$$\Delta_{LL} = \Delta_{Total} - \Delta_{DL} = 25.25 - 19.97 = 5.28 \text{ mm} < \frac{l_n}{360} = \frac{11,450}{360} = 31.80 \text{ mm} \ \textbf{\textit{o.k.}}$$ ***CSA A23.3-14 (Table 9.3)***

6.1.2. Simplified Superposition Procedure:

Reinforced Concrete Mechanics and Design – 1st Canadian edition – J. MacGregor (Section9-4)

For exterior span - service total load case:

$$M_o = \frac{w\times l_n^2}{8} = \frac{40.84\times11.45^2}{8} = 735.12 \text{ kN.m}$$

$$M_m = M_o + \frac{M_1}{2} + \frac{M_2}{2} = 735.12 - \frac{321.87}{2} - \frac{574.18}{2} = 292.48 \text{ kN.m}$$

$$\Delta_{Total} = \frac{5}{48}\times\frac{l_n^2}{E_c\times I_{e,avg}}\times\left(M_m + 0.1\times\left(M_1 + M_2\right)\right)$$

$$\Delta_{Total} = \frac{5}{48}\times\frac{11.45^2}{25,684\times3.35\times10^9}\times\left(292.48 - 0.1\times\left(321.87 + 574.18\right)\right) = 30.14 \text{ mm}$$

Where:

M_o = Simple span moment at midspan = $\dfrac{w\, l_n^2}{8}$

M_1 = Negative moment at the exterior support

M_2 = Negative moment at the interior support

Following the same procedure for dead service load level

$$\Delta_{DL} = 24.81 \text{ mm}$$

$$\Delta_{LL} = \Delta_{Total} - \Delta_{DL} = 30.14 - 24.81 = 5.33 \text{ mm} < \frac{l_n}{360} = \frac{11,450}{360} = 31.80 \text{ mm} \;\; \textbf{o.k.}$$ ***CSA A23.3-14 (Table 9.3)***

6.1.3. CSA Simplified Procedure: ***CAC Concrete Design Handbook- 4th Edition (6.3.1.1)***

This procedure is based on using Effective Moment of Inertia (I_e) and the midspan deflection may usually be used as an approximation of the maximum deflection:

$$\Delta = K\left(\frac{5}{48}\right)\frac{M \times l_n^2}{E_c \times I_e}$$ ***CAC Concrete Design Handbook- 4th Edition (6.3.1.1)***

$$K = 1.2 - 0.2 \times \frac{M_o}{M_m}$$ ***CAC Concrete Design Handbook- 4th Edition (Table 6.3(a))***

Where:

M_m = The net midspan moment

M_o = Simple span moment at midspan = $\frac{w\, l_n^2}{8}$

For exterior span - service total load case:

M_m = 292.48 kN.m

$$M_o = \frac{w \times l_n^2}{8} = \frac{40.84 \times (11.45)^2}{8} = 735.12 \text{ kN.m}$$

$$K = 1.2 - 0.2 \times \frac{735.12}{292.48} = 0.697$$

$$\Delta_{Total} = 0.697 \times \frac{5}{48} \times \frac{292.48 \times 11.45^2}{25,684 \times 3.35 \times 10^9} \times 10^6 = 32.40 \text{ mm}$$

Following the same procedure for dead service load level

$$\Delta_{DL} = 27.28 \text{ mm}$$

$$\Delta_{LL} = \Delta_{Total} - \Delta_{DL} = 32.40 - 27.28 = 5.12 \text{ mm} < \frac{l_n}{360} = \frac{11.45}{360} = 31.80 \text{ mm} \;\; \textbf{o.k.}$$ ***CSA A23.3-14 (Table 9.3)***

6.2. Time-Dependent (Long-Term) Deflections (Δ_{lt})

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) are estimated as follows. The results obtained from the PCA superposition procedure are used below, the same steps can be repeated to obtain the time-dependent deflections for the two other procedures.

$$\Delta_{cs} = \lambda_{\Delta} \times (\Delta_{sust})_{Inst}$$ ***PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)***

The total time-dependent (long-term) deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1+\lambda_{\Delta}) + [(\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst}]$$ ***CSA A23.3-04 (N9.8.2.5)***

Where:

$(\Delta_{sust})_{Inst} =$ Immediate (instantaneous) deflection due to sustained load, in.

$$\xi_s = \left[1 + \frac{s}{1+50\rho'}\right]$$ ***CSA23.3-14 (Eq .9.5)***

$(\Delta_{total})_{lt} =$ Time-dependent (long-term) total delfection, in.

$(\Delta_{total})_{Inst} =$ Total immediate (instantaneous) deflection, in.

For the exterior span

$\xi = 2$, consider the sustained load duration to be 60 months or more. ***CSA A23.3-14 (9.8.2.5)***

$\rho' = 0$, conservatively.

$$\lambda_{\Delta} = \frac{2}{1+50\times 0} = 2$$

$$\Delta_{cs} = 2\times 19.97 = 39.94 \text{ mm}$$

$$\Delta_{cs} + \Delta_{LL} = 39.94 + 5.28 \simeq 45.22 \text{ mm} \le \frac{l_n}{240} = \frac{11,450}{240} = 47.71 \text{ mm} \;\; \textbf{o.k.}$$ ***CSA A23.3-14 (Table 9.3)***

$$(\Delta_{total})_{lt} = 19.97\times(1+2)+(25.25-19.97) = 65.18 \text{ mm}$$

The previous deflection calculations are done considering a rectangular section in the negative moment regions. The same procedure can be followed to obtain the deflections values for the case where a T-shaped section is used in negative moment regions. Tables 9 and 10 in the comparison section show a results summary for both options.

7. Continuous Beam Analysis and Design – spBeam Software

spBeam is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per latest American (ACI 318-14) and Canadian (CSA A23.3-14) codes. spBeam can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, spBeam is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

spBeam provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

Beam analysis and design requires engineering judgment in most situations to properly simulate the behavior of the targeted beam and take into account important design considerations such as: designing the beam as rectangular or T-shaped sections; using the effective flange width or the center-to-center distance between the beam and the adjacent beams. Regardless which of these options is selected, spBeam provide users with options and flexibility to:

1. Design the beam as a rectangular cross-section or a T-shaped section.
2. Use the effective or full beam flange width.
3. Include the flanges effects in the deflection calculations.

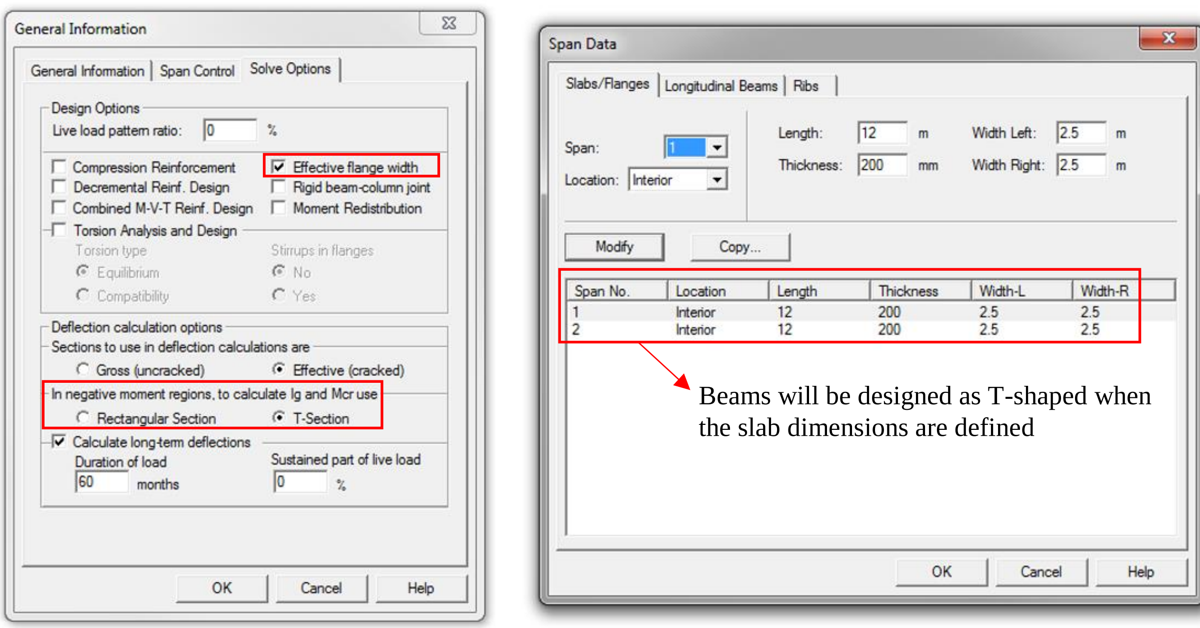
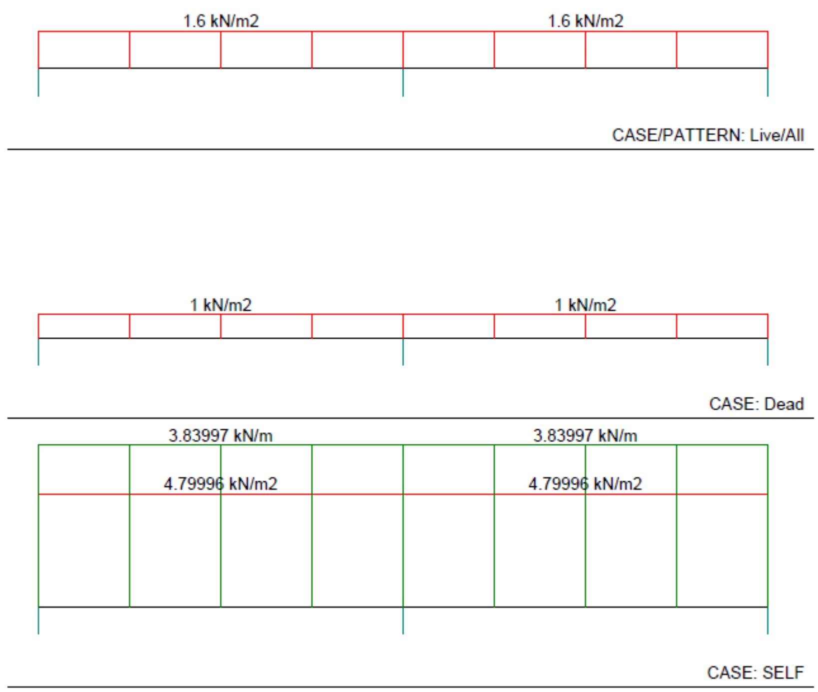


Figure 9 – spBeam Modeling and Solve Options

Also two other important options can be used:

4. Invoke moment redistribution to lower negative moments
5. Using gross (uncracked) or effective (cracked) moment of inertia

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the continuous beam on Grid 3 with effective flange width included in analysis, design and deflection calculations.



spBeam v5.00. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: C:\TSDA\spBeam\Grid3 - 1st floor.slb

Project: Continuous Beam Analysis and Design

Frame: Grid 3 - 1st floor

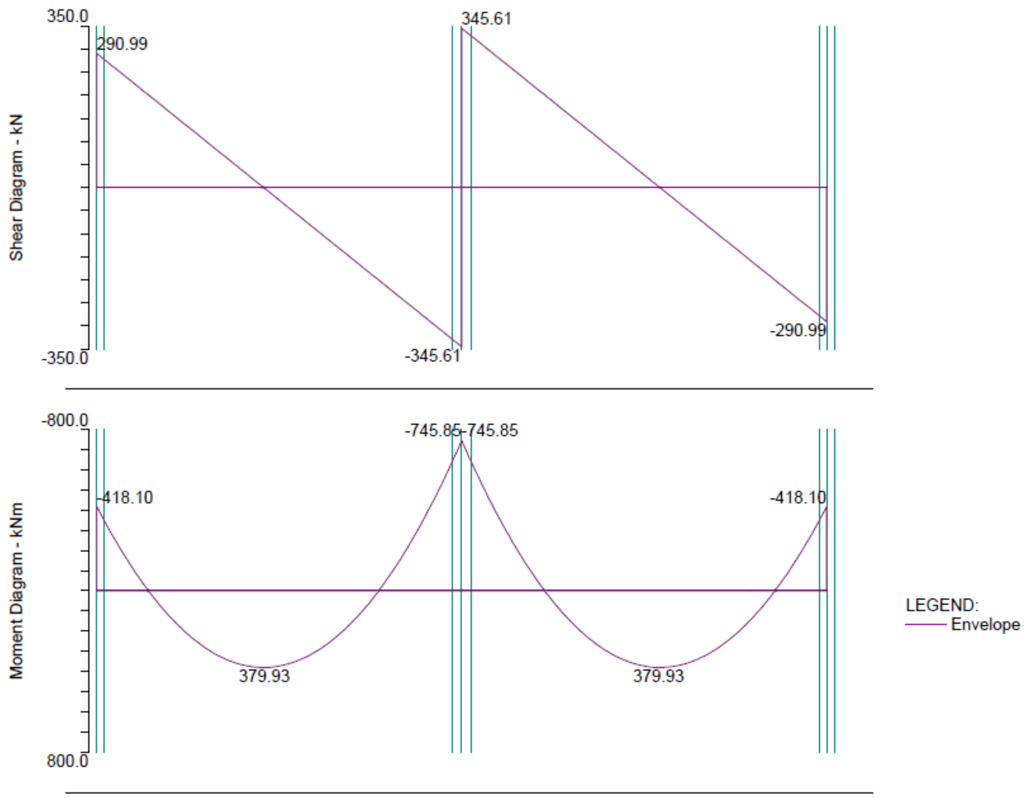
Engineer: SP

Code: CSA A23.3-14

Date: 07/16/18

Time: 08:55:02

Figure 10 – Loading (spBeam)



spBeam v5.00. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: C:\TSDA\spBeam\Grid3 - 1st floor.slb

Project: Continuous Beam Analysis and Design

Frame: Grid 3 - 1st floor

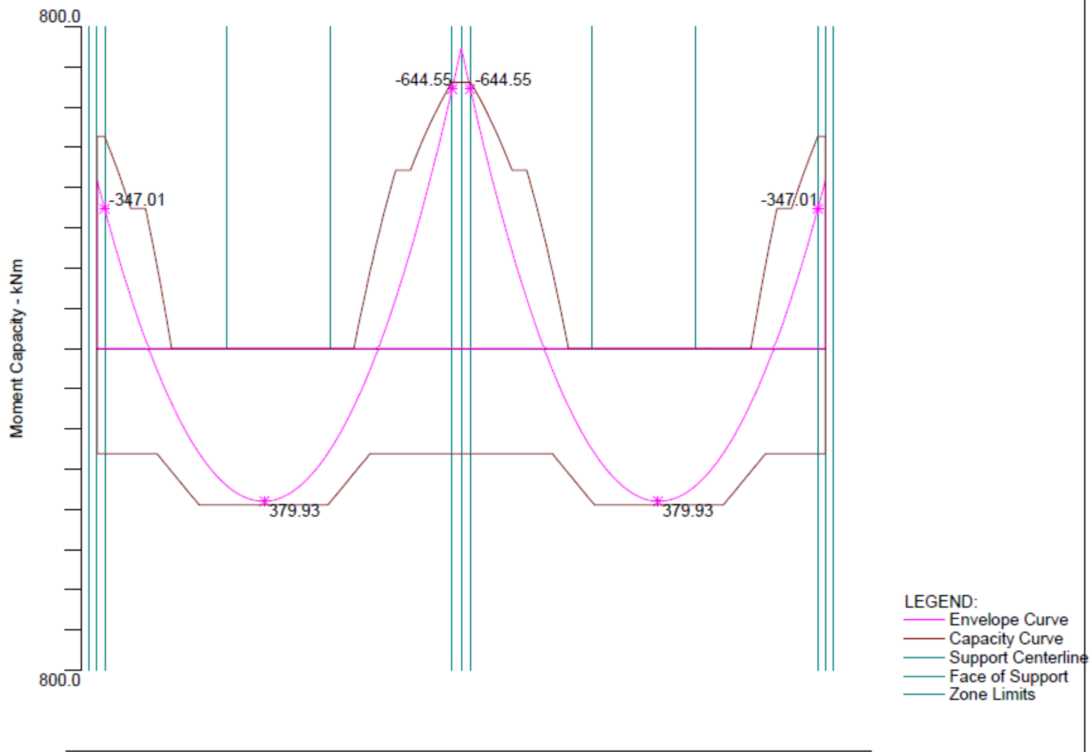
Engineer: SP

Code: CSA A23.3-14

Date: 07/16/18

Time: 08:55:46

Figure 11 – Internal Forces (spBeam)



spBeam v5.00. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: C:\TSDA\spBeam\Grid3 - 1st floor.slb

Project: Continuous Beam Analysis and Design

Frame: Grid 3 - 1st floor

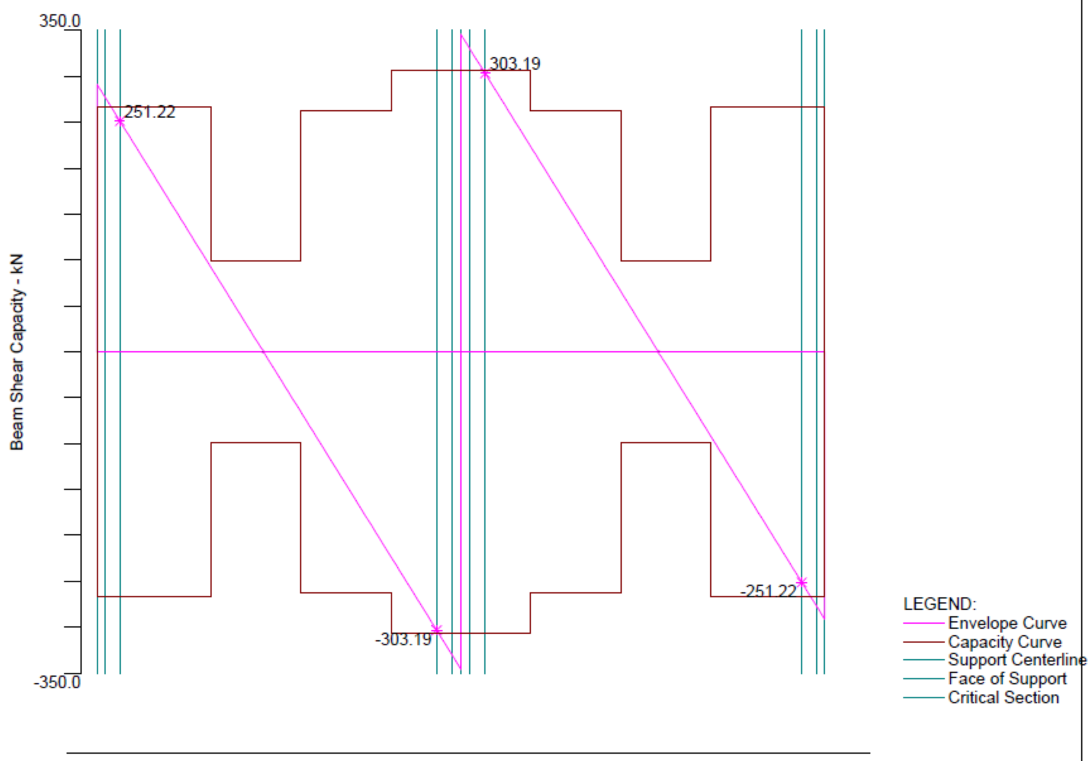
Engineer: SP

Code: CSA A23.3-14

Date: 07/16/18

Time: 08:56:25

Figure 12 – Moment Capacity Diagram (spBeam)



spBeam v5.00. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: C:\TSDA\spBeam\Grid3 - 1st floor.slb

Project: Continuous Beam Analysis and Design

Frame: Grid 3 - 1st floor

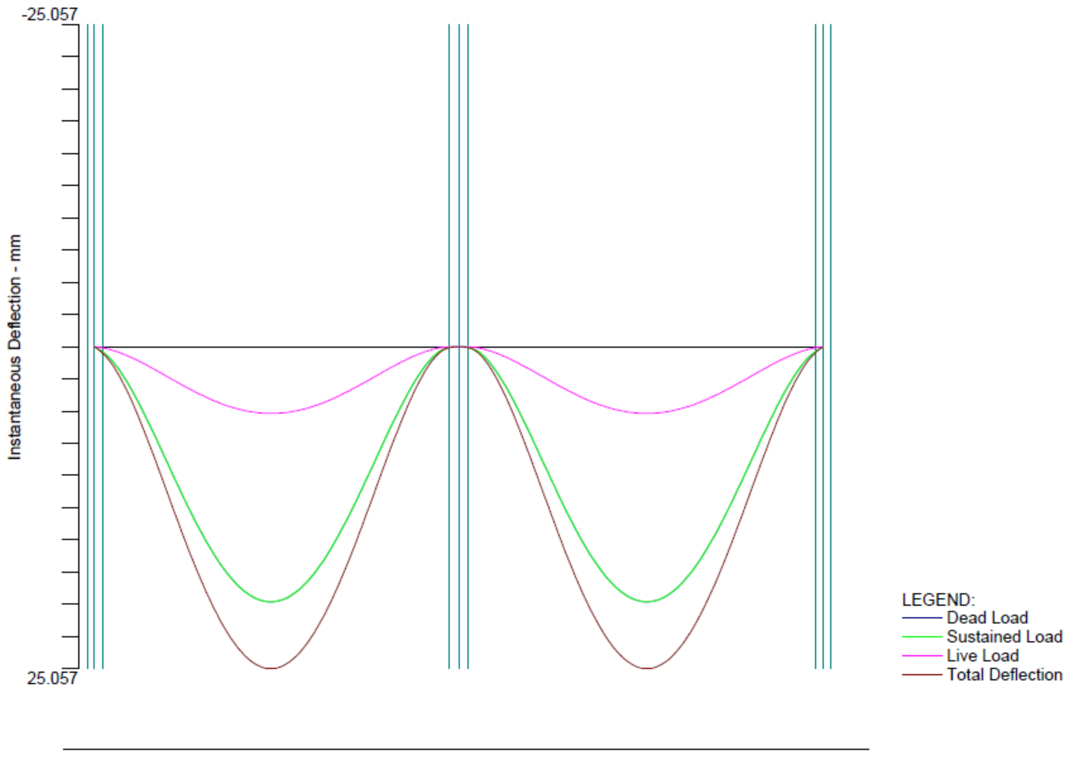
Engineer: SP

Code: CSA A23.3-14

Date: 07/16/18

Time: 08:56:51

Figure 13 – Shear Capacity Diagram (spBeam)



spBeam v5.00. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: C:\TSDA\spBeam\Grid3 - 1st floor.slb

Project: Continuous Beam Analysis and Design

Frame: Grid 3 - 1st floor

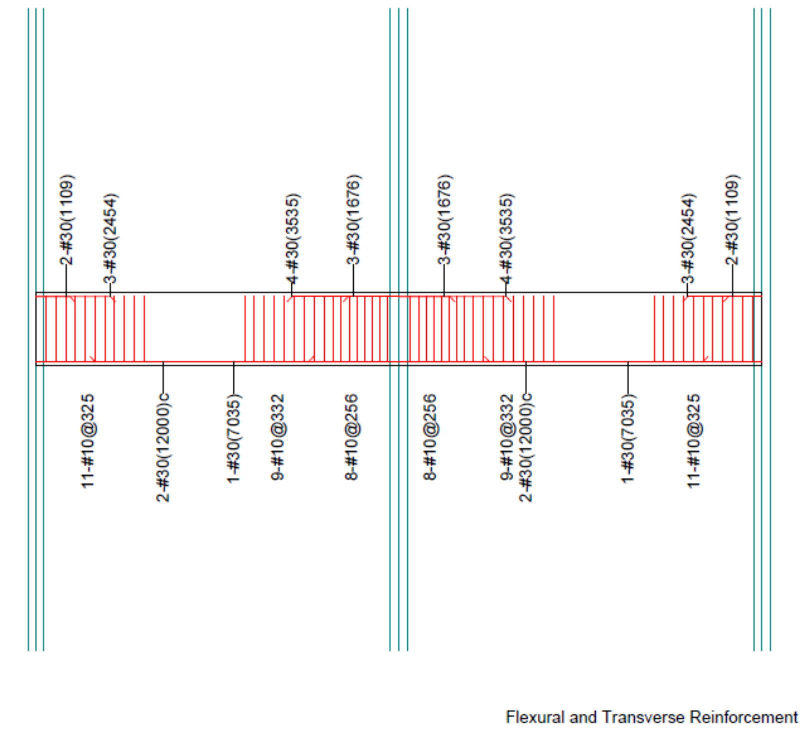
Engineer: SP

Code: CSA A23.3-14

Date: 07/16/18

Time: 08:57:11

Figure 14 – Immediate Deflection Diagram (spBeam)



spBeam v5.00. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: C:\TSDA\spBeam\Grid3 - 1st floor.slb

Project: Continuous Beam Analysis and Design

Frame: Grid 3 - 1st floor

Engineer: SP

Code: CSA A23.3-14

Date: 07/16/18

Time: 08:58:02

Figure 15 – Longitudinal and Lateral Reinforcement (spBeam)

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spBeam v5.00 © StructurePoint                                                    07-16-2018, 08:58:27 AM
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C:\TSDA\spBeam\Grid3 - 1st floor.slb                                                             Page 1

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                             oo   oooooo      oo    oo   ooooo     oo   oo    oo   oo   oo
                        o    oo   oo          oo    oo   oo    o   oo   oo    oo   oo   oo
                         ooooo    oo          ooooooo     ooooo     ooooo o   oo   oo   oo   (TM)
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spBeam v5.00 (TM)
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams and One-way Slab Systems

===

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[1] INPUT ECHO
==

General Information
===================
File name: C:\TSDA\spBeam\Grid3 - 1st floor.slb
Project: Continuous Beam Analysis and Design
Frame: Grid 3 - 1st floor
Engineer: SP
Code: CSA A23.3-14
Reinforcement Database: CSA G30.18
Mode: Design
Number of supports = 3
Floor System: One-Way/Beam

Live load pattern ratio = 0%
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
Combined M-V-T reinforcement design NOT selected.
Moment redistribution NOT selected.
Effective flange width calculations selected.
Rigid beam-column joint NOT selected.
Torsion analysis and design NOT selected.

Material Properties
===================

		Slabs|Beams	Columns	
wc	=	2447.3	2447.3	kg/m3
f'c	=	25	25	MPa
Ec	=	25684	25684	MPa
fr	=	1.5	3	MPa

Precast concrete construction is not selected.

fy	=	400	MPa, Bars are not epoxy-coated
fyt	=	400	MPa
Es	=	200000	MPa

Reinforcement Database
======================
Units: Db (mm), Ab (mm^2), Wb (kg/m)

Size	Db	Ab	Wb	Size	Db	Ab	Wb
#10	11	100	1	#15	16	200	2
#20	20	300	2	#25	25	500	4
#30	30	700	5	#35	36	1000	8
#45	44	1500	12	#55	56	2500	20

Span Data
=========

```
spBeam v5.00 © StructurePoint                                                                 07-16-2018, 08:58:27 AM
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C:\TSDA\spBeam\Grid3 - 1st floor.slb                                                                         Page 2

    Slabs
    -----
    Units: L1, wL, wR (m); t, bEff, Hmin (mm)
    Span Loc        L1        t       wL        wR     bEff      Hmin
    ---- ---- -------- -------- -------- -------- -------- --------
       1 Int    12.000      200    2.500     2.500     2800         0
       2 Int    12.000      200    2.500     2.500     2800         0

    Ribs and Longitudinal Beams
    ---------------------------
    Units: b, h, Sp (mm)
           ___________Ribs___________    ______Beams______   __Span__
    Span        b        h       Sp         b        h         Hmin
    ---- -------- -------- --------   -------- --------   --------
       1        0        0        0       400      600        636 *b
       2        0        0        0       400      600        636 *b
    NOTES:
    *b - Span depth is less than minimum. Deflection check required.

Support Data
============
    Columns
    -------
    Units: c1a, c2a, c1b, c2b (mm); Ha, Hb (m)
    Supp      c1a      c2a       Ha      c1b      c2b       Hb  Red%
    ---- -------- -------- --------  -------- -------- -------- ----
       1      500      500    4.200       500      500    4.400  100
       2      600      750    4.200       600      750    4.400  100
       3      500      500    4.200       500      500    4.400  100

    Boundary Conditions
    -------------------
    Units: Kz (kN/mm); Kry (kN-mm/rad)
    Supp    Spring Kz   Spring Kry Far End A Far End B
    ---- ------------ ------------ --------- ---------
       1            0            0   Fixed     Fixed
       2            0            0   Fixed     Fixed
       3            0            0   Fixed     Fixed

Load Data
=========
    Load Cases and Combinations
    ---------------------------
    Case     SELF     Dead     Live
    Type     DEAD     DEAD     LIVE
    ---- -------- -------- --------
    U1      1.250    1.250    1.500

    Area Loads
    ----------
    Units: Wa (kN/m2)
    Case/Patt Span            Wa
    --------- ---- ------------
    SELF         1         4.80
                 2         4.80
    Dead         1         1.00
                 2         1.00
    Live         1         1.60
                 2         1.60

    Line Loads
    ----------
    Units: Wa, Wb (kN/m), La, Lb (m)
    Case/Patt Span            Wa           La           Wb           Lb
    --------- ---- ------------ ------------ ------------ ------------
    SELF         1         3.84        0.000         3.84       12.000
                 2         3.84        0.000         3.84       12.000

Reinforcement Criteria
======================
    Slabs and Ribs
    --------------
                       _____Top bars___  __Bottom bars___
                          Min      Max      Min      Max
                       -------  -------  -------  -------
      Bar Size             #15      #15      #15      #15
      Bar spacing           25      457       25      457 mm
      Reinf ratio         0.14     5.00     0.14     5.00 %
      Cover                 20                20          mm
      There is NOT more than 300 mm of concrete below top bars.

    Beams
    -----
                       _____Top bars___  __Bottom bars___    _____Stirrups____
                          Min      Max      Min      Max        Min      Max
                       -------  -------  -------  -------    -------  -------
      Bar Size             #30      #30      #30      #30        #10      #10
```

```
     Bar spacing      25       457       25       457         152       457 mm
     Reinf ratio    0.14      5.00     0.14      5.00 %
     Cover            30                 30           mm
     Layer dist.      25                 25           mm
     No. of legs                                                2         6
     Side cover                                                20           mm
     1st Stirrup                                               76           mm
     There is NOT more than 300 mm of concrete below top bars.
```

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```

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[2] DESIGN RESULTS

Top Reinforcement

Units: Width (m), Mmax (kNm), Xmax (m), As (mm^2), Sp (mm)

Span	Zone	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Left	1.60	347.01	0.250	1500	4979	2093	376	5-#30 *5
	Midspan	1.60	0.00	5.975	0	4979	0	0	---
	Right	1.60	644.55	11.700	1500	4979	4692	251	7-#30
2	Left	1.60	644.55	0.300	1500	4979	4692	251	7-#30
	Midspan	1.60	0.00	6.025	0	4979	0	0	---
	Right	1.60	347.01	11.750	1500	4979	2093	376	5-#30 *5

NOTES:
*5 - Number of bars governed by maximum allowable spacing.

Top Bar Details

Units: Length (m)

	Left				Continuous		Right			
Span	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
1	3-#30	2.45	2-#30*	1.11	---		4-#30	3.53	3-#30*	1.68
2	4-#30	3.53	3-#30*	1.68	---		3-#30	2.45	2-#30*	1.11

NOTES:
* - Bar cut-off location shall be manually checked for compliance with CSA A23.3, 11.2.13.

Top Bar Development Lengths

Units: Length (mm)

	Left				Continuous		Right			
Span	Bars	Length	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
1	3-#30	859.31	2-#30	859.31	---		4-#30	1376.20	3-#30	1376.20
2	4-#30	1376.20	3-#30	1376.20	---		3-#30	859.31	2-#30	859.31

Bottom Reinforcement

Units: Width (m), Mmax (kNm), Xmax (m), As (mm^2), Sp (mm)

Span	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	0.40	379.93	5.517	600	23619	2048	152	3-#30
2	0.40	379.93	6.483	600	23619	2048	152	3-#30

Bottom Bar Details

Units: Start (m), Length (m)

	Long Bars			Short Bars		
Span	Bars	Start	Length	Bars	Start	Length

1	2-#30	0.00	12.00	1-#30*	1.97	7.03
2	2-#30	0.00	12.00	1-#30*	3.00	7.03

NOTES:
* - Bar cut-off location shall be manually checked for compliance with CSA A23.3, 11.2.13.

Bottom Bar Development Lengths

Units: DevLen (mm)

	Long Bars		Short Bars	
Span	Bars	DevLen	Bars	DevLen
1	2-#30	1401.49	1-#30	1401.49
2	2-#30	1401.49	1-#30	1401.49

Flexural Capacity

Units: x (m), As (mm^2), PhiMn, Mu (kNm)

		Top						Bottom					
Span	x	AsTop	PhiMn-	Mu-	Comb	Pat	Status	AsBot	PhiMn+	Mu+	Comb	Pat	Status
1	0.000	3500	-526.44	-418.10	U1	All	---	1400	261.14	0.00	U1	All	---
	0.250	3500	-526.44	-347.01	U1	All	OK	1400	261.14	0.00	U1	All	OK
	1.109	2100	-348.04	-128.03	U1	All	OK	1400	261.14	0.00	U1	All	OK
	1.595	2100	-348.04	-21.56	U1	All	OK	1400	261.14	0.00	U1	All	OK
	1.968	1189	-208.83	0.00	U1	All	OK	1400	261.14	51.79	U1	All	OK
	2.454	0	0.00	0.00	U1	All	OK	1643	305.82	136.20	U1	All	OK
	3.369	0	0.00	0.00	U1	All	OK	2100	389.41	261.14	U1	All	OK
	4.257	0	0.00	0.00	U1	All	OK	2100	389.41	339.98	U1	All	OK
	5.517	0	0.00	0.00	U1	All	OK	2100	389.41	379.93	U1	All	OK
	6.000	0	0.00	0.00	U1	All	OK	2100	389.41	372.86	U1	All	OK
	7.601	0	0.00	0.00	U1	All	OK	2100	389.41	261.14	U1	All	OK
	7.692	0	0.00	0.00	U1	All	OK	2054	381.09	250.71	U1	All	OK
	8.465	0	0.00	0.00	U1	All	OK	1668	310.49	144.30	U1	All	OK
	9.003	1093	-193.20	0.00	U1	All	OK	1400	261.14	51.69	U1	All	OK
	9.842	2800	-442.60	-123.51	U1	All	OK	1400	261.14	0.00	U1	All	OK
	10.324	2800	-442.60	-241.06	U1	All	OK	1400	261.14	0.00	U1	All	OK
	11.700	4900	-661.94	-644.55	U1	All	OK	1400	261.14	0.00	U1	All	OK
	11.760	4900	-661.94	-664.43	U1	All	---	1400	261.14	0.00	U1	All	---
	12.000	4900	-661.94	-745.85	U1	All	---	1400	261.14	0.00	U1	All	---
2	0.000	4900	-661.94	-745.85	U1	All	---	1400	261.14	0.00	U1	All	---
	0.240	4900	-661.94	-664.43	U1	All	---	1400	261.14	0.00	U1	All	---
	0.300	4900	-661.94	-644.55	U1	All	OK	1400	261.14	0.00	U1	All	OK
	1.676	2800	-442.60	-241.06	U1	All	OK	1400	261.14	0.00	U1	All	OK
	2.158	2800	-442.60	-123.51	U1	All	OK	1400	261.14	0.00	U1	All	OK
	2.997	1093	-193.20	0.00	U1	All	OK	1400	261.14	51.69	U1	All	OK
	3.535	0	0.00	0.00	U1	All	OK	1668	310.49	144.30	U1	All	OK
	4.307	0	0.00	0.00	U1	All	OK	2054	381.09	250.71	U1	All	OK
	4.399	0	0.00	0.00	U1	All	OK	2100	389.41	261.14	U1	All	OK
	6.000	0	0.00	0.00	U1	All	OK	2100	389.41	372.86	U1	All	OK
	6.483	0	0.00	0.00	U1	All	OK	2100	389.41	379.93	U1	All	OK
	7.742	0	0.00	0.00	U1	All	OK	2100	389.41	339.98	U1	All	OK
	8.631	0	0.00	0.00	U1	All	OK	2100	389.41	261.14	U1	All	OK
	9.546	0	0.00	0.00	U1	All	OK	1643	305.82	136.20	U1	All	OK
	10.032	1189	-208.83	0.00	U1	All	OK	1400	261.14	51.79	U1	All	OK
	10.405	2100	-348.04	-21.56	U1	All	OK	1400	261.14	0.00	U1	All	OK
	10.891	2100	-348.04	-128.03	U1	All	OK	1400	261.14	0.00	U1	All	OK
	11.750	3500	-526.44	-347.01	U1	All	OK	1400	261.14	0.00	U1	All	OK
	12.000	3500	-526.44	-418.10	U1	All	---	1400	261.14	0.00	U1	All	---

Longitudinal Beam Transverse Reinforcement Demand and Capacity

Section Properties

Units: dv (mm), Av/s (mm^2/mm), PhiVc, Vrmax (kN)

Span	dv	(Av/s)min	PhiVc	Vrmax
1	499.5	0.300	116.89	811.76
2	499.5	0.300	116.89	811.76

Beam Transverse Reinforcement Demand

Units: Start, End, Xu (mm), Vu (m), Av/s (kN/mm^2)

			Required				Demand
Span	Start	End	Xu	Vu	Comb/Patt	Av/s	Av/s
1	0.326	2.243	0.750	251.22	U1/All	0.554	0.554
	2.243	3.736	2.243	172.02	U1/All	0.227	0.300 *8
	3.736	5.229	3.736	92.82	U1/All	0.000	0.000
	5.229	6.721	6.721	65.59	U1/All	0.000	0.000
	6.721	8.214	8.214	144.79	U1/All	0.115	0.300 *8
	8.214	9.707	9.707	223.99	U1/All	0.442	0.442
	9.707	11.624	11.200	303.19	U1/All	0.768	0.768

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      2     0.376    2.293    0.800   303.19    U1/All    0.768    0.768
            2.293    3.786    2.293   223.99    U1/All    0.442    0.442
            3.786    5.279    3.786   144.79    U1/All    0.115    0.300 *8
            5.279    6.771    5.279    65.59    U1/All    0.000    0.000
            6.771    8.264    8.264    92.82    U1/All    0.000    0.000
            8.264    9.757    9.757   172.02    U1/All    0.227    0.300 *8
            9.757   11.674   11.250   251.22    U1/All    0.554    0.554

   NOTES:
   *8 - Minimum transverse (stirrup) reinforcement governs.

   Beam Transverse Reinforcement Details
   -------------------------------------
   Units: spacing & distance (mm).
   Span Size Stirrups (2 legs each unless otherwise noted)
   ---- ---- ---------------------------------------------
      1  #10 11 @ 325 + <-- 2986 --> + 9 @ 332 + 8 @ 256
      2  #10 8 @ 256 + 9 @ 332 + <-- 2986 --> + 11 @ 325

   Beam Transverse Reinforcement Capacity
   --------------------------------------
   Units: Start, End, Xu (m), Vu, PhiVn (kN), Av/s (mm^2/mm), Av (mm^2), Sp (mm)
                              __________________Required__________________  _____________Provided____________
   Span    Start      End         Xu        Vu Comb/Patt     Av/s Reqd/Min        Av       Sp     Av/s    PhiVn
   ---- -------- -------- -------- -------- --------- -------- -------- -------- -------- -------- --------
      1    0.000    0.326    0.750   251.22    U1/All    -----    -----    -----    -----    -----    -----
           0.326    3.736    0.750   251.22    U1/All    0.554     1.85    200.0      325    0.616   266.30
           3.736    6.721    3.736    92.82    U1/All    0.000     0.00    -----    -----    -----    99.61
           6.721    9.707    9.707   223.99    U1/All    0.442     1.47    200.0      332    0.603   263.12
           9.707   11.624   11.200   303.19    U1/All    0.768     2.56    200.0      256    0.783   306.76
          11.624   12.000   11.200   303.19    U1/All    -----    -----    -----    -----    -----    -----

      2    0.000    0.376    0.800   303.19    U1/All    -----    -----    -----    -----    -----    -----
           0.376    2.293    0.800   303.19    U1/All    0.768     2.56    200.0      256    0.783   306.76
           2.293    5.279    2.293   223.99    U1/All    0.442     1.47    200.0      332    0.603   263.12
           5.279    8.264    8.264    92.82    U1/All    0.000     0.00    -----    -----    -----    99.61
           8.264   11.674   11.250   251.22    U1/All    0.554     1.85    200.0      325    0.616   266.30
          11.674   12.000   11.250   251.22    U1/All    -----    -----    -----    -----    -----    -----

Slab Shear Capacity
===================
   Units: b, dv (mm), Xu (m), PhiVc, Vu(kN)
   Span        b       dv     Beta   Vratio         PhiVc           Vu           Xu
   ---- -------- -------- -------- -------- ------------ ------------ ------------
      1 --- Not checked ---
      2 --- Not checked ---

Material Takeoff
================
   Reinforcement in the Direction of Analysis
   ------------------------------------------
   Top Bars:      315.9 kg    <=>  13.16 kg/m    <=>  2.633 kg/m^2
   Bottom Bars:   341.1 kg    <=>  14.21 kg/m    <=>  2.842 kg/m^2
   Stirrups:       80.9 kg    <=>   3.37 kg/m    <=>  0.674 kg/m^2
   Total Steel:   737.9 kg    <=>  30.75 kg/m    <=>  6.149 kg/m^2
   Concrete:       27.8 m^3   <=>   1.16 m^3/m   <=>  0.232 m^3/m^2
```

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```


[3] DEFLECTION RESULTS

Section Properties

Frame Section Properties

Units: Ig, Icr (mm^4), Mcr (kNm)

Span	Zone	M+ve Ig	M+ve Icr	M+ve Mcr	M-ve Ig	M-ve Icr	M-ve Mcr
1	Left	1.5200e+010	2.2385e+009	52.62	7.2000e+009	4.4766e+009	-36.00
	Midspan	1.5200e+010	2.8726e+009	52.62	7.2000e+009	0.00000	-36.00
	Right	1.5200e+010	2.2385e+009	52.62	7.2000e+009	5.6286e+009	-36.00
2	Left	1.5200e+010	2.2385e+009	52.62	7.2000e+009	5.6286e+009	-36.00
	Midspan	1.5200e+010	2.8726e+009	52.62	7.2000e+009	0.00000	-36.00
	Right	1.5200e+010	2.2385e+009	52.62	7.2000e+009	4.4766e+009	-36.00

NOTES: M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Frame Effective Section Properties

Units: Ie, Ie,avg (mm^4), Mmax (kNm)

			Load Level					
			Dead		Sustained		Dead+Live	
Span	Zone	Weight	Mmax	Ie	Mmax	Ie	Mmax	Ie
1	Middle	0.850	235.19	3.0107e+009	235.19	3.0107e+009	292.48	2.9444e+009
	Right	0.150	-461.71	5.6293e+009	-461.71	5.6293e+009	-574.18	5.6290e+009
	Span Avg	----	----	3.4035e+009	----	3.4035e+009	----	3.3471e+009
2	Left	0.150	-461.71	5.6293e+009	-461.71	5.6293e+009	-574.18	5.6290e+009
	Middle	0.850	235.19	3.0107e+009	235.19	3.0107e+009	292.48	2.9444e+009
	Span Avg	----	----	3.4035e+009	----	3.4035e+009	----	3.3471e+009

Instantaneous Deflections

Extreme Instantaneous Frame Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

				Live			Total	
Span	Direction	Value	Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	19.86	---	5.20	5.20	19.86	25.06
		Loc	5.860	---	5.860	5.860	5.860	5.860
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
2	Down	Def	19.86	---	5.20	5.20	19.86	25.06
		Loc	6.140	---	6.140	6.140	6.140	6.140
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---

Long-term Deflections

Long-term Deflection Factors

Time dependant factor for sustained loads = 2.000
Units: Astop, Asbot (mm^2), b, d (mm), Rho' (%), Lambda (-)

Span	Zone	M+ve Astop	M+ve b	M+ve d	M+ve Rho'	M+ve Lambda	M-ve Asbot	M-ve b	M-ve d	M-ve Rho'	M-ve Lambda
1	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000

NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Extreme Long-term Frame Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

Span	Direction	Value	cs	cs+lu	cs+l	Total
1	Down	Def	39.72	44.92	44.92	64.78
		Loc	5.860	5.860	5.860	5.860
	Up	Def	---	---	---	---
		Loc	---	---	---	---
2	Down	Def	39.72	44.92	44.92	64.78
		Loc	6.140	6.140	6.140	6.140
	Up	Def	---	---	---	---
		Loc	---	---	---	---

NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

8. Design Results Comparison and Conclusions

The following tables show the comparison between hand results and spBeam model results.

<table>
<tr><th colspan="7">Table 5 - Comparison of Moments and Flexural Reinforcement</th></tr>
<tr><th rowspan="2">Location</th><th colspan="2">M_f^*, kN.m</th><th colspan="2">$A_{s,req}$, mm^2</th><th colspan="2">Reinforcement</th></tr>
<tr><th>Hand</th><th>spBeam</th><th>Hand</th><th>spBeam</th><th>Hand</th><th>spBeam</th></tr>
<tr><td>Exterior Negative</td><td>418.1</td><td>418.1</td><td>2,092</td><td>2,093</td><td>5 – 30M</td><td>5 – 30M</td></tr>
<tr><td>Positive</td><td>372.9</td><td>380.0</td><td>2,014</td><td>2,048</td><td>3 – 30M</td><td>3 – 30M</td></tr>
<tr><td>Interior Negative</td><td>745.9</td><td>745.8</td><td>4,692</td><td>4,692</td><td>7 – 30M</td><td>7 – 30M</td></tr>
<tr><td colspan="7">* negative moments are taken at the faces of supports</td></tr>
</table>

<table>
<tr><th colspan="9">Table 6 - Comparison of Shear and lateral Reinforcement</th></tr>
<tr><th rowspan="2">Location</th><th colspan="2">V_f^*, kN</th><th colspan="2">$(A_v/s)_{req}$, mm^2/mm</th><th colspan="2">Reinforcement</th><th colspan="2">V_r, kN</th></tr>
<tr><th>Hand</th><th>spBeam</th><th>Hand</th><th>spBeam</th><th>Hand</th><th>spBeam</th><th>Hand</th><th>spBeam</th></tr>
<tr><td>Interior Negative</td><td>253.9</td><td>251.2</td><td>0.78</td><td>0.77</td><td>10M @ 250 mm</td><td>10M @ 256 mm</td><td>305.9</td><td>306.8</td></tr>
<tr><td colspan="9">* Shear values are taken at distance d_v from the faces of supports</td></tr>
</table>

<table>
<tr><th colspan="9">Table 7 - Comparison of Section Properties (<u>Rectangular</u> Negative Moment Regions)</th></tr>
<tr><th rowspan="3">Location</th><th colspan="2">I_{cr}, mm^4 (×10^9)</th><th colspan="6">$I_{e,avg}$, mm^4 (×10^9)</th></tr>
<tr><th rowspan="2">Hand</th><th rowspan="2">spBeam</th><th colspan="3">Hand</th><th colspan="3">spBeam</th></tr>
<tr><th>DL</th><th>DL+LL$_{sus}$</th><th>Total</th><th>DL</th><th>DL+LL$_{sus}$</th><th>Total</th></tr>
<tr><td>Exterior Negative</td><td>7.20</td><td>7.20</td><td rowspan="3">3.40</td><td rowspan="3">3.40</td><td rowspan="3">3.35</td><td rowspan="3">3.40</td><td rowspan="3">3.40</td><td rowspan="3">3.35</td></tr>
<tr><td>Positive</td><td>15.20</td><td>15.20</td></tr>
<tr><td>Interior Negative</td><td>7.20</td><td>7.20</td></tr>
</table>

<table>
<tr><th colspan="9">Table 8 - Comparison of Section Properties (<u>T-shaped</u> Negative Moment Regions)</th></tr>
<tr><th rowspan="3">Location</th><th colspan="2">I_{cr}, mm^4 (×10^9)</th><th colspan="6">$I_{e,avg}$, mm^4 (×10^9)</th></tr>
<tr><th rowspan="2">Hand</th><th rowspan="2">spBeam</th><th colspan="3">Hand</th><th colspan="3">spBeam</th></tr>
<tr><th>DL</th><th>DL+LL$_{sus}$</th><th>Total</th><th>DL</th><th>DL+LL$_{sus}$</th><th>Total</th></tr>
<tr><td>Exterior Negative</td><td>4.48</td><td>4.48</td><td rowspan="3">3.44</td><td rowspan="3">3.44</td><td rowspan="3">3.37</td><td rowspan="3">3.44</td><td rowspan="3">3.44</td><td rowspan="3">3.37</td></tr>
<tr><td>Positive</td><td>2.87</td><td>2.87</td></tr>
<tr><td>Interior Negative</td><td>5.63</td><td>5.63</td></tr>
</table>

Table 9 - Comparison of Maximum Deflection (**Rectangular** Negative Moment Regions)		
Procedure	Δ_{inst}, mm	Δ_{lt}, mm
spBeam	25.06	64.78
PCA Superposition	25.25 (+0.8%)	65.18
Simplified Superposition	32.41 (+29.3 %)	86.97
PCA Simplified	31.38 (+25.2 %)	81.01

Table 10 - Comparison of Maximum Deflection (**T-shaped** Negative Moment Regions)		
Procedures	Δ_{inst}, mm	Δ_{lt}, mm
spBeam	24.93	64.28
PCA Superposition	29.50 (+18.3 %)	75.92
Simplified Superposition	32.22 (+29.2 %)	86.19
PCA Simplified	31.20 (+25.1 %)	80.29

The results of all the hand calculations used illustrated above are in agreement with the automated exact results obtained from the spBeam program except for deflection (see section 8.2 for detailed discussion).

8.1. Deflection calculation methods:

Deflections calculations in reinforced concrete structures can be very tedious and time consuming because of the difficulty of accounting for the actual end boundary conditions in a building frame. As a result, numerous methods to estimate the deflection and the member stiffness have been presented in literature. It is important to note that these methods can only estimate deflections within an accuracy range of 20% to 40% as can be seen in Tables 9 and 10. It is important for the designer to be aware of this broad range of accuracy, especially in the modeling, design, and detailing of deflection-sensitive members.

spBeam uses elastic frame analysis (stiffness method) to obtain deflections along the beam span by discretizing the beam span into 110 elements. It also takes into account the adjacent spans effects, shape effects, supporting members stiffnesses above and below the beam, and cracked section effects based on the applied forces. This level of detail provides the maximum accuracy possible compared with other approximate methods used to calculate deflections.

In tables 9 and 10, the deflection value calculated by spBeam is lower than all the other values calculated by the approximate methods. This can be expected since approximate methods have a built-in conservatism to accommodate a wide range of applications and conditions. The designer can use spBeam and exploit its numerous features to get a closer deflection estimate and optimize the depth and size of the beam or slab under consideration.

9. Boundary condition effects on continuous beam deflections

Boundary conditions can have significant effects on the behavior of continuous beams especially deflections. This section focuses on investigating boundary conditions commonly found in building and used with continuous beams. In each section we will explore when to use each condition, and how to properly model them using spBeam.

The following discussion provides recommendations for each of the grids in the building floor system under consideration, observations on the impact of the condition, and the proper modeling method in spBeam.

9.1. Beam supported by columns- Grid 3

The length and size of the columns above and below the beam are modeled allowing spBeam to calculate the rotational stiffness for each member at the joint to correctly determine the beam end moments.

In the support date, the stiffness % of 100 indicates full utilization of the column and beam geometry. Whereas a value of 999 will impose a fixed support condition and a value of zero will impose a pinned support condition ignoring the supporting columns entirely.

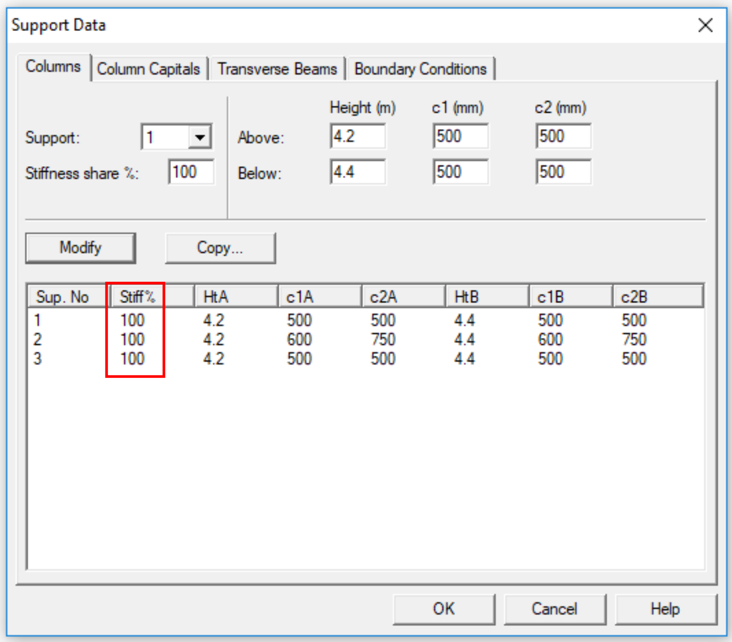


Figure 16 – Beam Supported by Columns – Defining Supports Geometry (spBeam)

9.2. Beam supported by transverse beams - Grid 4

To model transverse beam as a support, the cross-sectional dimensions and length **should not** be entered as shown below since this tab is reserved for two-way slab systems with beams between all supports.

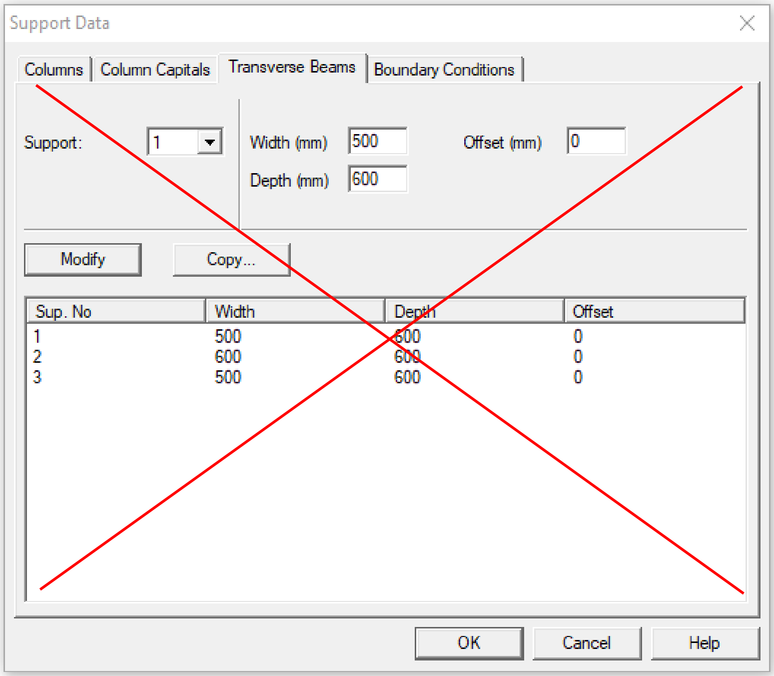


Figure 17 – Beam Supported by Transverse Beams – Defining Supports Geometry for two-way Slab systems (spBeam)

Alternately, the designer should enter a value for the rotational stiffness for the supporting transverse beams calculated by an external method if desired and to provide additional flexibility for special cases as the boundary conditions tab shown below.

The CSA code approximates the moment at transverse beam end support to 2/3 the moment at column end support. This can be used as a reference to find the rotational stiffness of transverse beams by trial and error.

CSA A23.3-14 (Table 9.1)

For Grid 4, adding the approximated rotational stiffnesses shown in the following Figure will result in (276 kN.m) negative moment at the end support which is 2/3 the negative moment at the end support for Grid 3 (418.1 kN.m).

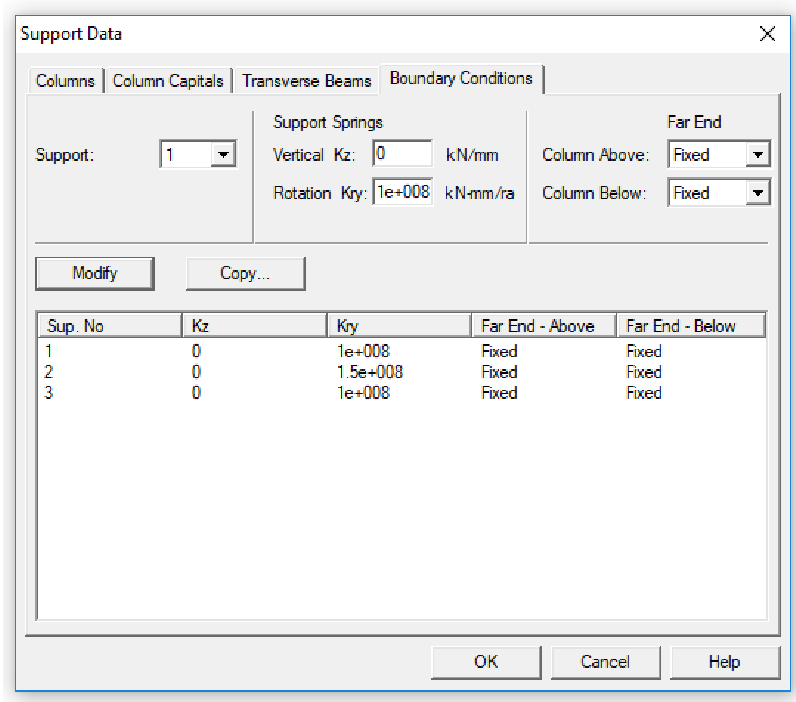


Figure 18 – Beam Supported by Transverse Beams – Defining Supports Stiffnesses (spBeam)

spBeam designs the beam for the moment at the center of the support when a rotation stiffness is defined. To design the beam for moments at the face of the supporting transverse beam, a dummy column can be defined with all parameters equals to zero except c_1 where it should be set equal to the width of the transverse beam. spBeam will obtain shear and moment values at critical sections and use them for design.

In Grid 2, a combination of the boundary conditions shown in grids 3 and 4 should be utilized where the end columns are modeled for support 1 and 3 and a transverse beam is modeled for support 2 as shown below.

9.3. Beam supported by transverse walls - Grid 5

In this model the beam is cast monolithically with the shear wall, the wall define as an elongated column.

9.4. Beam supported by masonry bearing walls - Grid C

The wall can be modeled as pin support with a stiffness percent of zero.

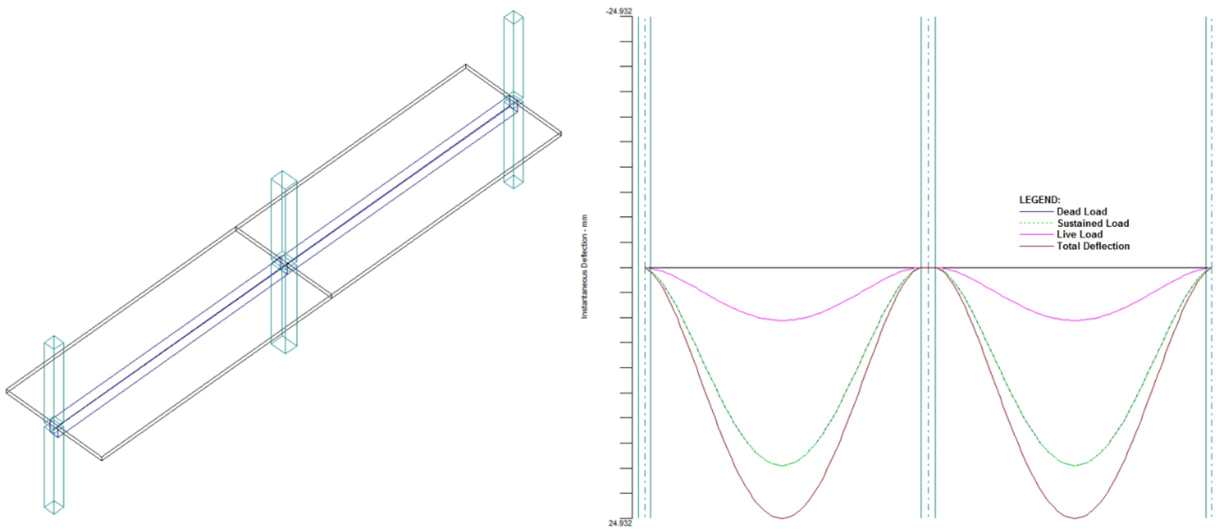
9.5. Beam supported by longitudinal walls - Grids 1, 6, and A

The beam can be modeled up to the face of the wall. At the face of the wall the beam should be restrained by a fixed support and the wall width can be ignored.

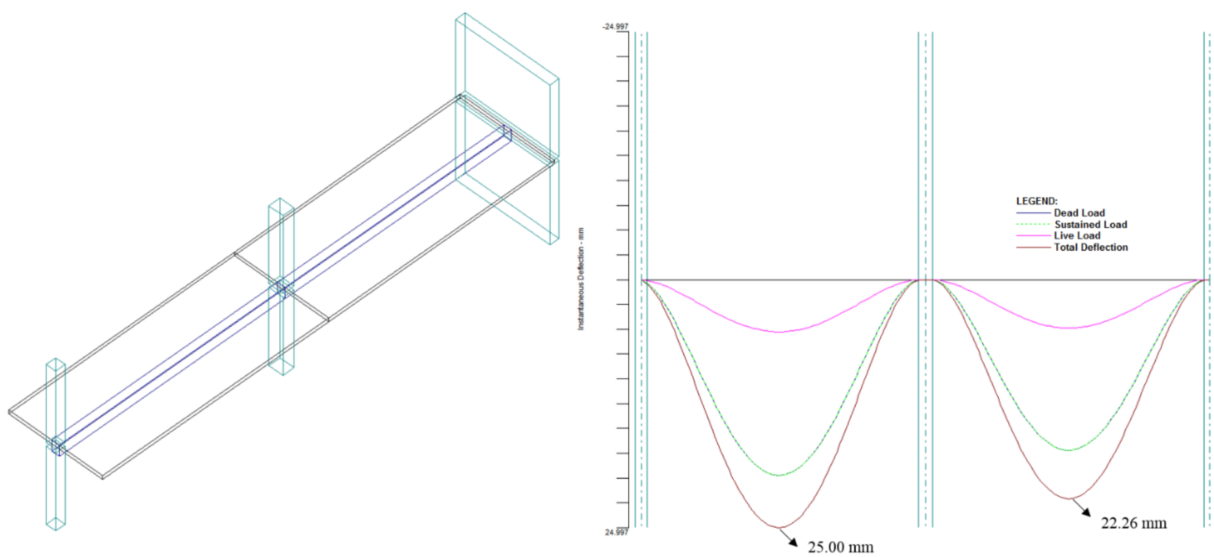
To illustrate the effects of boundary conditions, beams along grids 2, 3, 4 and 5 in Figure 1 are modeled using spBeam and deflections obtained from these models are compared.

Table 12 - Comparison of Continuous Beams with Different Boundary Conditions (Deflections)				
Grid #	$(\Delta_{inst})_{total}$, mm	Δ_{cs}, mm	Δ_{lt}, mm	Notes
3	24.93	39.35	64.28	All columns are modeled with the exact geometry
5	22.26 (-10.7 %)	34.9 (-11.3 %)	57.06 (-11.2 %)	Transverse shear wall is modeled with the exact geometry for the right span
2	34.68 (+39.1 %)	54.14 (+37.6 %)	88.82 (+38.2 %)	The middle support is modeled as transverse beam using rotational stiffness with dummy column to obtain shear and moment values at the critical sections.
4	40.38 (+62.0%)	63.59 (+61.6 %)	103.97 (+61.7 %)	All supports are modeled as transverse beams using rotational stiffness with dummy columns to obtain shear and moment values at the critical sections.

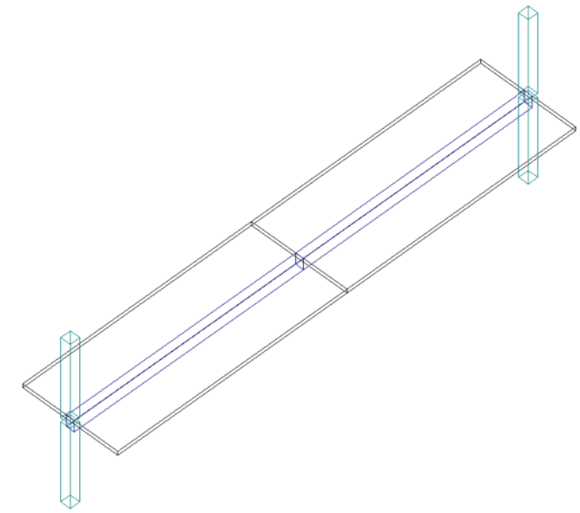
Beam along Grid 3

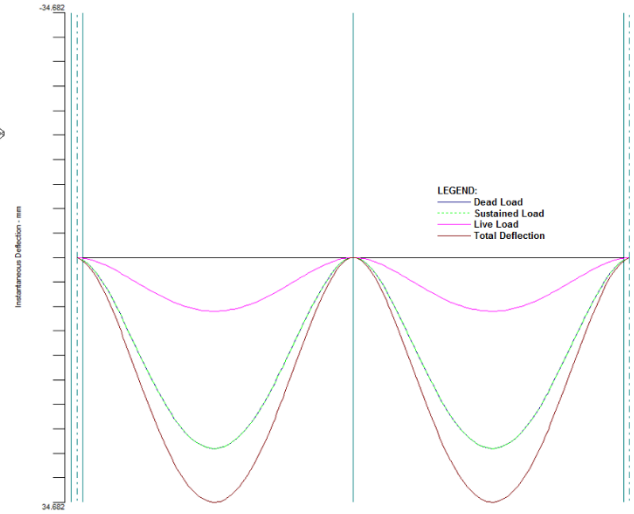


Beam along Grid 5



Beam along Grid 2





Beam along Grid 4